## LABORATORY 3: Phasors, Complex Power, and Filters

Note: If your partner is no longer in the class, please talk to the instructor.

## Material covered:

- Phasors
- Complex power, real power, reactive power
- First Order Filters
- Multi-stage Circuits
- Bode Plots
- Filter Design Problem


## Part A: Phasors

Overview Notes:
Impedance revisited

Resistors:


Inductors


## Capacitors



Time domain


Phasor form

| Component | Time Domain | Phasor Form |
| :---: | :---: | :---: |
| Resistor | $\mathrm{V}_{\mathrm{R}}(\mathrm{t})=\mathrm{I}_{\mathrm{R}}(\mathrm{t}) \mathrm{R}$ | $\mathrm{VR}(\mathrm{t})=\mathrm{I}_{\mathrm{R}}(\mathrm{t}) \mathrm{R}$ |
| Inductor | $\mathrm{V}_{\mathrm{L}}(\mathrm{t})=\mathrm{Ld} \mathrm{I}_{\mathrm{L}}(\mathrm{t}) / \mathrm{dt}$ | $\mathrm{V}_{\mathrm{L}}(\mathrm{t})=(\mathrm{j} \omega \mathrm{L}) \mathrm{I}_{\mathrm{L}}(\mathrm{t})$ |
| Capacitor | $\mathrm{I}_{\mathrm{C}}(\mathrm{t})=\mathrm{Cd} \mathrm{V}_{\mathrm{C}}(\mathrm{t}) / \mathrm{dt}$ | $\mathrm{V}_{\mathrm{C}}(\mathrm{t})=\mathrm{I}_{\mathrm{C}}(\mathrm{t}) /(\mathrm{j} \omega \mathrm{C})$ |

Recall, circuit analysis using impedance is identical to that with resistors. The only difference is the numbers are now complex.

Complex notation: Recall, we can express a sinusoidal signal in the time domain as as $V(t)=A \cos (\omega t+\phi)$ where A is the amplitude of the signal, $\omega$ is the radial frequency and $\varphi$ is the phase shift. The equivalent signal in phasor notation is written as $V=A \angle \phi$ where the $\omega t$ term is implied. Alternatively, polar notation can be used $V=A e^{i \phi}$ where the real part of the polar expansion is the measureable
signal (voltage, current). We can also use the rectangular form for complex numbers, $V=A_{R}+j A_{I}$, where $\mathrm{A}_{\mathrm{R}}$ and $\mathrm{A}_{\mathrm{I}}$ are the real and imaginary components, respectively. The relationships between phasor and rectangular form can be written as

| Phasor | Rectangular |
| :--- | :--- |
| Magnitude: $A=\left(A_{R}^{2}+A_{I}^{2}\right)^{\frac{1}{2}}$ | Real: $A_{R}=A \cos (\phi)$ |
| Phase: $\phi=\tan ^{-1}\left(\frac{A_{I}}{A_{R}}\right)$ | Imaginary: $A_{I}=A \sin (\phi)$ |

## Measurements:



The above circuit is implemented using the Vsin component as the source component. Using a source voltage of 5 V at 5 kHz , a plot of the source voltage and the capacitor voltage are shown below.


One of the issues with LTSpice is that it automatically considers sources as 0 for $\mathrm{t}<0$, which introduces a transient component. In the case of AC analysis, the LTSpice calculations are effectively $\mathrm{V}_{\mathrm{o}} \sin (\omega \mathrm{t}) \mathrm{u}(\mathrm{t})$. To avoid including the transient effects in our calculations, we want to start our plots after the transients have attenuated to zero. Typically, several periods will be sufficient. In the above case, a start time of

1 ms corresponds to 5 periods after $\mathrm{t}=0$, which is sufficient to ignore the effects of the initial transients. In this example the run time is $1 \mathrm{E}-4$ to $1.6 \mathrm{E}-4 \mathrm{~s}$, with a maximum step size of $1 \mathrm{E}-6 \mathrm{~s}$, allowing 3 full cycles to be displayed. The start time, run time, and step size values depend on the frequency and circuit. Amplitude:

Measured Amplitude (Instrumentation Board and LTSpice): In the case of sinusoidal sources, the magnitude of the voltage is equivalent to the amplitude of the sine wave. Remember, amplitude is half the peak to peak voltage. You can estimate it from the plots or use the measurement tab. It is clear from the plot on the previous page that the amplitude is 5 . As we have experienced, the Discovery Board amplitude for the same source voltage setting will probably be a little different.

Calculated Amplitude: The calculated amplitude is determined by applying impedance analysis to the circuit, similar to what was seen in Unit 2. Again, considering the RC circuit and using a voltage divider, we can derive the voltage across the capacitor as

$$
\begin{aligned}
& V_{C}=\rightarrow \frac{Z_{C}}{Z_{C}+Z_{R}}\left(V_{0} \angle \phi\right)=\frac{\frac{1}{j \omega C}}{\frac{1}{j \omega C}+R}\left(V_{0} \angle \phi\right) \\
& \left|V_{c}\right|=\left|\frac{\frac{1}{j \omega C}}{\frac{1}{j \omega C}+R}\left(V_{0} \angle \phi\right)\right| \rightarrow\left|\frac{1}{1+j \omega R C}\left(V_{0} \angle \phi\right)\right|=\left|\frac{1}{j(2 \pi 5 E 3)(1 E 3)(1 E-7)+1} 5\right|=1.5
\end{aligned}
$$

Phase: The phase of the waveform is defined as the angle relative to some reference. Typically, the source waveform has zero phase and is used as the reference.

Instrumentation Board (and LTSpice): Recognizing that one period corresponds to $2 \pi\left(360^{\circ}\right)$, the phase shift can be determined by the time difference between the two peaks. Since the source frequency is 5 kHz , one period occurs every $200 \mu \mathrm{~s}$. In seconds, $\Delta \mathrm{t}$, the phase shift of the capacitor voltage can be measured as the time between when the source peak occurs and the capacitor voltage peak occurs. Using the cursors and recording the $\Delta \mathrm{t}$, we see a delay (negative $\Delta \mathrm{t}$ ) of $40 \mu$ s for the capacitor voltage. The phase shift is
then $\left(\frac{\text { phase shift }}{\text { period }}\right)(360)=\left(\frac{-40}{200}\right)(360)=-72^{\circ}$. Note, the capacitor voltage peak
occurs after the source peak, indicating a negative phase shift in the capacitor voltage relative to the source. Again, in both Discovery Board and LTSpice, you should use cursors to make these measurements.
Calculated Phase: The calculated phase of the waveform is determined using the phasor form of the complex numbers

$$
\begin{aligned}
& V_{C}=\frac{1}{j \omega R C+1} 5 \angle 0=\frac{1}{j(2 \pi 5 E 3)(1 E 3)(1 E-7)+1} 5 \angle 0=\frac{1}{j 3.14+1} 5 \angle 0=\frac{1 \angle 0^{\circ}}{\sqrt{10.85} \angle 72^{\circ}} 5 \angle 0 \\
& \angle V_{C}=\angle 0^{\circ}+\angle 0^{\circ}-\angle 72^{\circ}=-72^{\circ}
\end{aligned}
$$

Laboratory:
RC Circuits:


Implement the above circuit. Set the source amplitude to 2Vpp sinusoid (1V amplitude) with a 0 V DC offset.
a. For a frequency of 2750 Hz , calculate the phasor form of the voltage across the capacitor.
b. Implement the circuit and obtain the phase and magnitude from the Discovery Board.
c. Implement the circuit in LTSpice and obtain phase and magnitude from the simulation. Use the Vsin component for the source.
d. Compare your above results and determine a time domain expression for voltage across the capacitor.
e. Repeat a. and b. for a frequency of 27850 Hz . At this frequency would you characterize the capacitor as a short circuit? an open circuit? neither?
f. Repeat $a$. and $b$. for a frequency of 50 Hz . At this frequency would you characterize the capacitor as a short circuit? an open circuit? neither?
Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

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B.2:

RL Circuits:


Implement the above circuit. Set the source amplitude to 0.2 Vpp sinusoid with a 0 V DC offset.
a. For a frequency of 2750 Hz , calculate the phasor form of the voltage across the inductor.
b. Implement the circuit and obtain the phase and magnitude from the Discovery Board.
c. Implement the circuit in LTSpice and obtain phase and magnitude from the simulation.
d. Compare your above results and determine a time domain expression for voltage across the inductor.
e. Repeat a. and b. for a frequency of 27500 Hz . At this frequency would you characterize the inductor as a short circuit? an open circuit? neither?
f. Repeat a . and b . for a frequency of 50 Hz . At this frequency would you characterize the inductor as a short circuit? an open circuit? neither?
Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

## PART A: Proof of Concepts list <br> A.1: RC Circuits - Prove how the capacitor impedance changes with frequency using phasor analysis (phasor form) <br> A.2: RL Circuits - Prove how the inductor impedance changes with frequency using phasor analysis (phasor form)

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## Part B: Complex Power

Complex power, $\mathrm{S}[\mathrm{VA}$ ], is defined as the sum of the real power, P [W], and the reactive power, Q [VAR].

$$
S=P+j Q
$$

Only resistors consume real power and P must be greater than zero. Inductors store reactive power, with Q being positive for inductors and negative for capacitors.


When considering an arbitrary impedance with current Vo and Io, we can use one of the three representations to calculate complex power in phasor form as:

1) $S=V_{\text {RUS }} I_{\text {RuS }}^{*}=\left|V_{\text {RuS }} \| I_{\text {RUS }}\right|(\angle V-\angle I)$
2) $S=\left|I_{\text {RuS }}\right|^{2} Z=\left|I_{\text {RUS }}\right|^{2}(\angle Z)$ (should be multiplied by absolute value of $Z$ )
3) $S=\frac{\left|V_{\text {Rus }}\right|^{2}}{Z^{*}}=\frac{\left|V_{\text {Rus }}\right|^{2}}{|Z|}(\angle Z)$

Any of the above equations can be used to find the complex power. Recall, Euler's law can be used to put the expression in rectangular form to find the real and reactive components.

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## B.1. Phasor Analysis and Complex Power



Implement the above circuit. (The above diagram was made in PSpice but you should be able to simulate it in LTSPice). Set the source amplitude to 5 V amplitude with a frequency of 795 Hz .
a. Use phasor analysis to calculate the current through Rs
b. Using the first power expression, determine the complex power produced by the source. Recall, the current through Rs is equal to the current through Vs since those components are in series.
c. Using any of the power expressions, determine the complex power in Rs, Z1 and Z2.
d. Using your instrumentation board, measure the voltage across Rs and use Ohm's Law to determine the current through Rs. Based on this measurement, calculate the power produced.
e. Measure the voltage across Rs, Z 1 and Z 2 and use the third expression to determine the power for each.
f. Complete the following table and compare your results.

|  | Calculated power |  | Measured power |  |
| :---: | :---: | :---: | :---: | :---: |
|  | P | jQ | P | jQ |
| Rs |  |  |  |  |
| Z 1 |  |  |  |  |
| $\mathrm{Z2}$ |  |  |  |  |
| Vs |  |  |  |  |

Include these charts in your Proof of Concept report and relevant screen shots for source and at least one load.

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A.2.


Implement the above circuit. (The above diagram was made in PSpice but you should be able to simulate it in LTSpice). Set the source amplitude to 5 V amplitude with a frequency of 795 Hz . Your capacitor value does not have to be exact. Try to get something reasonably close.
a. Repeat the previous calculations and measurements for the above circuit.
b. What do you notice when compare these two circuits? Be sure to answer this in your discussion

|  | Calculated power |  | Measured power |  |
| :---: | :---: | :---: | :---: | :---: |
|  | P | jQ | P | jQ |
| Rs |  |  |  |  |
| Z 1 |  |  |  |  |
| Z 2 |  |  |  |  |
| $\mathrm{Z3}$ |  |  |  |  |
|  |  |  |  |  |
| Vs |  |  |  |  |

Include these charts in your Proof of Concept report and relevant screen shots for source and at least one load.

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PART B: Proof of Concepts list
B.1: Prove that the power produced by the source balances the power consumed by the load(s).
B.2: Prove that the power produced by the source balances the power consumed by the load(s).

## Part C: Filters and Bode Plots

## C.1. RL Filter Circuit Calculations


.ac dec 100 1Hz 1E6Hz

Determine the transfer function for the voltage across the resistor,
$H_{V R}(s)=$ $\qquad$

Determine the transfer function for the voltage across the inductor,
$H_{V L}(s)=$ $\qquad$

What is the cutoff frequency for this circuit?

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As the frequency approaches zero (DC), the voltage across the inductor approaches

As the frequency approaches infinity, the voltage across the inductor approaches

When measuring the voltage across the inductor, is this circuit a low pass filter or a high pass filter?

## C.2. RL Filter Circuit Simulation and Experiment

Build the circuit. Set your source amplitude to 2 Vpp (1V amplitude) with a 0 V offset and adjust the frequency as indicated in the table. Measure the output voltage amplitude and phase across the inductor for various frequencies and fill in the following table with your calculations, LTSpice, Instrumentation Board measurements.

| Inductor |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Magnitude |  |  | Phase [Degrees] |  |  |
| Freq. <br> [Hz] | Rad. <br> Freq. <br> [rad/s] | Calculated | LTSpice | Measured | Calculated | LTSpice | Measured |
| 47.7 |  |  |  |  |  |  |  |
| 159 |  |  |  |  |  |  |  |
| 477 |  |  |  |  |  |  |  |
| $\mathrm{f}_{\mathrm{c}}$ |  |  |  |  |  |  |  |
| 1590 |  |  |  |  |  |  |  |
| 4770 |  |  |  |  |  |  |  |
| 15.9k |  |  |  |  |  |  |  |

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Include this table of results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs! If there are any significant differences between measured and calculated results, comment on possible reasons. (Recall the characteristics of a real inductor.)

For your above measured results, determine the magnitude in dB and the phase of the transfer function. When calculating the transfer function magnitude, remember to divide to divide the output magnitude by the source amplitude (which in this case is 1 ). The phase can be obtained directly from your measured results in the previous table.

| Radial <br> Frequency <br> $[\mathrm{rad} / \mathrm{s}]$ | $\log (\omega)$ | $20 \log \|\mathrm{H}(\mathrm{s})\|[\mathrm{dB}]$ | Phase <br> $\|\mathrm{H}(\mathrm{s})\|$ |
| :---: | :--- | :--- | :--- |
| 300 |  |  |  |
| 1 E 3 |  |  |  |
| 3 E 3 |  |  |  |
| $\omega_{\mathrm{c}}$ |  |  |  |
| 1 E 4 |  |  |  |
| 3 E 4 |  |  |  |
| 1 E 5 |  |  |  |

Include this table of results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

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Using your measured results, generate a $\log -\log$ plot $(\mathrm{dB}-\log (\omega))$ of the magnitude of the transfer function vs. frequency. Also, plot the angle (phase) of the transfer function against $\log (\omega)$. For the same circuit, perform an AC Sweep in LTSpice and compare your results. Can you find a way to get this information using the Analog Discovery Board or M1K or M2K board directly? How?

Magnitude


Is the cutoff frequency a -3 dB point?
Does your stopband display a $20 \mathrm{~dB} /$ decade drop?

Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

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## Phase



What is the change of phase between $\omega=300[\mathrm{rad} / \mathrm{s}]$ and $\omega=5 \mathrm{E} 4[\mathrm{rad} / \mathrm{s}]$ ? Is your result consistent with expectations from the transfer function?

Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

PART C: Proof of Concepts list
C.1: Prove that gain and phase change with frequency for an RL circuit.
C.2: Prove that the Bode Plot for gain and phase reflects this change of frequency for an RL circuit.

## Part D: Second Order Filters and Bode Plots

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## Overview Notes:

Two stage circuits:


The above figure represents a two stage circuit. Recall, the transfer function relates the ratio the input to the output of a stage, $\frac{V_{\text {out }}(s)}{V_{\text {in }}(s)}=H(s)$. In the above figure, recognize that $\mathrm{V}_{\text {out1 }}=\mathrm{V}_{\text {in2 }}$. The transfer function can be applied to each stage. Applying the transfer function to each stage we can derive the equation, $V_{\text {out } 2}(s)=H_{2}(s) V_{\text {in } 2}(s)=H_{2}(s) V_{\text {out } 1}(s)=H_{2}(s) H_{1}(s) V_{\text {in } 1}(s)$. Finally, the relationship between $\mathrm{V}_{\text {out } 2}$ and $\mathrm{V}_{\text {in } 1}$ can be written as $\frac{V_{\text {out } 2}(s)}{V_{\text {in1 }}(s)}=H_{2}(s) H_{1}(s)$. This equation is the product of the two transfer functions. By designing each stage to produce a particular circuit response, the final output can be designed to meet a specific goal.

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## Filter types:




Highpass Filter

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Bandpass Filter


Notch/Bandstop Filter

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## D1: Multi-stage Filter Circuit Calculations



Determine the transfer function for the voltage across the resistor, R4.
$\mathrm{H}(\mathrm{s})=$ $\qquad$ (symbolically)

This circuit is which of the following?
highpass filter, lowpass filter, bandpass filter, notch filter What are the zeros
of the transfer function? $\qquad$

What are the poles of the transfer function? $\qquad$

What is the gain of the passband, in dB ?

What is the slope of the stopband(s) in $\mathrm{dB} /$ decade? $\qquad$

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## Multi-stage Filter Circuit Simulation and Experiment

Build the circuit and measure the output voltage for various frequencies. Your source amplitude should 200 mV . Your Oscilloscope V/div should be $0.5 \mathrm{~V} /$ div or less. Fill in the following table with your calculations and Analog Discovery Board measurements. Remember to scale your measured output voltage with your input voltage to obtain the transfer function.

|  | Magnitude, $\|\mathrm{H}(\mathrm{s})\|$ |  |  |
| :---: | :--- | :--- | :--- |
| Frequency <br> $[\mathrm{Hz}]$ | Calculated | Measured | 20log $\|\mathrm{H}(\mathrm{s})\|$ (use <br> measured <br> value) |
| 15.9 |  |  |  |
| 48 |  |  |  |
| 159 |  |  |  |
| 488 |  |  |  |
| 1590 |  |  |  |

Using your measured results, generate a Bode plot of the magnitude. Compare the measured results to your analytic expression and LTSpice simulation.

Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

PART D: Proof of Concepts list
D.1: Prove the expected Magnitude Bode plot for the multistage circuit.

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## Part E: Filter Design Problem

Design a lowpass filter that meets the following specifications.

1. Cutoff frequency: 1.59 kHz
2. 60 dB rolloff in the stopband
3. A single unity gain opamp
4. $|\mathrm{H}(\mathrm{j} \omega)| \boldsymbol{\rightarrow}-3 \mathrm{~dB}$ relative to the passband at the cutoff frequency. (Remember a triple pole has a -9 dB correction relative to the straight line approximation. You will need to underdamp a second order circuit.)
5. Use L and C component values found in your kit. You can use resistors found on the center table in the laboratory room.

Some flexibility exists in meeting the specifications. In design problems, perfection is not usually possible, but deviations should be small. If you don't quite meet specification, explain why and explain what you would do to fix the problem.

## E. 1 Filter Design Calculations

Determine the transfer function that meets the above specifications
$\mathrm{H}(\mathrm{s})=$ $\qquad$ (symbolically)

Draw the circuit, labeling the component values

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## E.1. Filter Design Simulation and Calculations

Build the circuit and verify that the specifications are met by taking measurements at appropriate frequencies. You should base your Vin amplitude around that setting.

|  | Magnitude, $\|\mathrm{H}(\mathrm{s})\|$ |  |  |
| :---: | :--- | :--- | :--- |
| Radial <br> Frequency <br> [rad/s] | Calculated | Measured <br> $\mid$ Vout//Vin $\mid$ | 20log $\|\mathrm{H}(\mathrm{s})\|$ (use <br> measured <br> value) |
| 100 |  |  |  |
| 300 |  |  |  |
| 1 E 3 |  |  |  |
| 3 E 3 |  |  |  |
| 9 E 3 |  |  |  |
| 10 E 3 |  |  |  |
| 15 E 3 |  |  |  |
| 20 E 3 |  |  |  |
| 30 E 3 |  |  |  |
| 100 E 3 |  |  |  |

Due to the steep rolloff and noise effects, you may have difficulty obtaining data as you move further into the stopband.

For the same circuit, perform an AC Sweep in LTSpice and compare your results.

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Include screen shots of your results in your Proof of Concept Report. There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

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PART E: Proof of Concepts list
Prove that your designed filter meets specifications.
```


## Alpha Lab Applications (Extra Credit....) Passive filter vs.

## Active filter

1. What is the difference between a passive filter and an active filter?
2. Find three different active filter configurations. How are they different?
3. Describe any conditions for which an active filter is much more useful than a passive filter.
4. Simulate any active filter configuration. Calculate and compare.

Include screen shots of your results in your Proof of Concept Report. You do not need to build the circuit. It is optional.

## SUMMARY of Concepts

Concept List that must be accounted for in your Proof of Concepts

## PART A:

1. RC Circuits - Prove how the capacitor impedance changes with frequency using phasor analysis (phasor form)
2. RL Circuits - Prove how the inductor impedance changes with frequency using phasor analysis (phasor form)
PART B:
3. Prove that the power produced by the source balances the power consumed by the load(s).
4. Prove that the power produced by the source balances the power consumed by the load(s).
PART C:
5. Prove that gain and phase change with frequency for an RL circuit.
6. Prove that the Bode Plot for gain and phase reflects this change of frequency for an RL circuit.
PART D:
7. Prove the expected Magnitude Bode plot for the multistage circuit.

## PART E:

1. Prove that your designed filter meets specifications.

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