Phasor-Only State Estimation

Syncrhonized Phasor Measurements Tutorial

Dr. Luigi Vanfretti
Email: vanfretti@alum.rpi.edu
Web Page: http://web.me.com/vanfretti/

Electric Power Systems Division

The Royal Institute of Technology
Stockholm, Sweden

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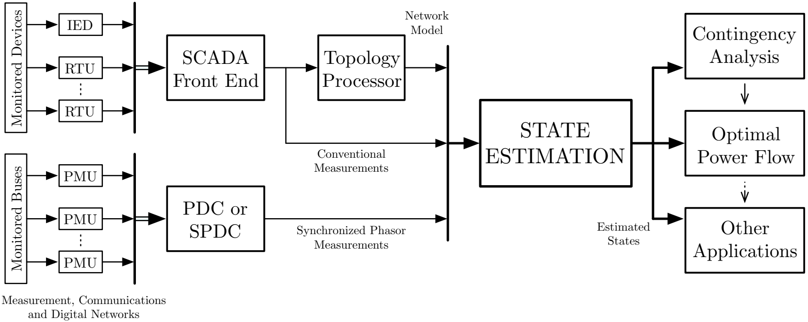
Outline

- Introduction
- Phadke's & Thorp's Linear SE
- Angular Errors in Phasor Data and Modeling in Polar Coordinates
- The Phasor State Estimator
- Measurement Model in Polar Coordinates
- Least Squares Formulation and Successive Solution Algorithms
- Extension for Phase Angle Shift Correction
- Observability and Redundancy
- Application to AEPs HV Network
- Conclusions

State Estimation and EMS Systems Today

⇒ SCADA - conventional measurements
⇒ PDC - phasor measurements
⇒ Topology Processor - network model from status info.
⇒ Observability analysis - determines feasible solution with given measurements, identifies unobservable branches and observable islands
⇒ Bad data processing - determines errors in the data, elliminates bad data given enough redundancy
⇒ SE process - provides estimated states of the system → $\boldsymbol{V}$, $\boldsymbol{\theta}$, transformer tap, generator settings, and power flows in branches and loads.



PMUs are included to reflect recent adoption in some control centers

Conventional WLS State Estimation - I

⇒ The relationship between the system state $\boldsymbol{x}$, and the measurements $\boldsymbol{z}$ is given by

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{e} \tag{1}$$

where $\boldsymbol{e}$ is the vector of measurement errors, and $\boldsymbol{h}(\boldsymbol{x})$ is the nonlinear function which is formed by

- power flow equations → for measured $\boldsymbol{P}$ & $\boldsymbol{Q}$ injections and $\boldsymbol{S}_{ij}$ flows, and
- $I_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2}/V_i$ → for line current flow measurements ($\boldsymbol{I}_{ij}$)

⇒ In a power system with m measurements, assuming independent errors ($E\left\{e_i e_j\right\} = 0$), a WLS solution is obtained by minimizing

$$J(\boldsymbol{x}) = \sum_{i=1}^{m} \frac{\left(z_i - h_i(\boldsymbol{x})\right)^2}{R_{ii}} = \left[\boldsymbol{x} - \boldsymbol{h}(\boldsymbol{x})\right]^T \boldsymbol{R}^{-1} \left[\boldsymbol{x} - \boldsymbol{h}(\boldsymbol{x})\right] \tag{2}$$

where $\boldsymbol{R}$ is a diagonal matrix of covariances σ_i^2, $i = 1, 2, \ldots, m$

⇒ Solution to (2) is iterative using Newton Methods [1, 2].

Conventional WLS State Estimation - II

⇒ Linearizing $\boldsymbol{h}(\boldsymbol{x})$ about $\boldsymbol{x}^{(0)}$ and using only the first-order term of the Taylor series results in the Gauss-Newton iterative solution method

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^{k} - \left[\boldsymbol{G}(\boldsymbol{x}^{k})\right]^{-1}\left[-\boldsymbol{H}^{T}(\boldsymbol{x}^{k})\boldsymbol{R}^{-1}\left(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}^{k})\right)\right] \quad (3)$$

where
k is the iteration index,
$\boldsymbol{x}^{k}$ is the solution vector at iteration k ,
$\boldsymbol{H}(\boldsymbol{x}^{k})$ is the measurement Jacobian matrix, $\boldsymbol{H}(\boldsymbol{x}) = \left[\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right]$, evaluated at iteration k, and
$\boldsymbol{G}(\boldsymbol{x}^{k})$ is the *gain* matrix, $\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{H}^{T}(\boldsymbol{x})\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{x})$, evaluated at iteration k.

⇒The gain matrix is not inverted, instead it is decomposed by triangular factorization and solved using backward substitutions at each iteration, hence

$$\boldsymbol{G}(\boldsymbol{x}^{k})\left(\boldsymbol{x}^{k+1} - \boldsymbol{x}^{k}\right) = \boldsymbol{H}^{T}(\boldsymbol{x}^{k})\boldsymbol{R}^{-1}\left(\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}^{k})\right) \quad (4)$$

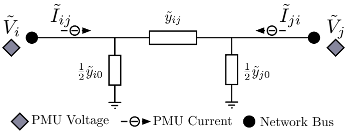
is solved iteratively until a certain tolerance, $\left|\boldsymbol{x}^{k+1} - \boldsymbol{x}^{k}\right| < \epsilon$, is satisfied. Note that any measurement error is distributed among all the states.

▸ Additional Slides at the end of the presentation summarize several approaches used to include PMU data into conventional state estimators.

Phadke and Thorp's Linear State Estimator [4] - I

A. G. Phadke, J. S. Thorp, and K. J. Karimi, "State Estimation with Phasor Measurements," IEEE Transactions on Power Systems, vol. 1, no. 1, pp. 233-238, Feb. 1986.

⇒ Measure bus voltage and line current *phasors* using PMUs, and **formulate the estimation problem in terms of *complex* voltages and currents** - rectangular coordinate formulation.



$$\begin{aligned} \tilde{I}_{ij} &= (\tilde{y}_{ij} + \tilde{y}_{i0})\tilde{V}_i - \tilde{y}_{ij}\tilde{V}_j \\ \tilde{I}_{ji} &= -\tilde{y}_{ij}\tilde{V}_i + (\tilde{y}_{ij} + \tilde{y}_{j0})\tilde{V}_j \end{aligned} \quad (5)$$

Define a bus-measurement admittance matrix $\tilde{\boldsymbol{Y}}$ (only including the admittances of branches with PMU-measurements)

$$\tilde{\boldsymbol{Y}} = \begin{bmatrix} \tilde{y}_{ij} + \tilde{y}_{i0} & -\tilde{y}_{ij} \\ -\tilde{y}_{ij} & \tilde{y}_{ij} + \tilde{y}_{j0} \end{bmatrix} \quad (6)$$

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{y}}\boldsymbol{A}^T + \tilde{\boldsymbol{y}}_{\boldsymbol{S}}$$

$\boldsymbol{A}$: current measurement-bus incidence matrix $(m \times b)$ — m=rows = # $\tilde{I}_m$,b = cols = #$\tilde{V}_m$,
$\tilde{\boldsymbol{y}}$: diagonal primitive matrix of series admittances of metered elements $(m \times m)$,
$\tilde{\boldsymbol{y}}_{\boldsymbol{S}}$: shunt primitive matrix of shunt admittances metered ends $(m \times b)$.

The complex currents can be written as

$$\tilde{\boldsymbol{I}} = \left(\tilde{\boldsymbol{y}}\boldsymbol{A}^T + \tilde{\boldsymbol{y}}_{\boldsymbol{S}}\right)\tilde{\boldsymbol{V}} \quad (7)$$

Phadke and Thorp's Linear State Estimator [4] - II

⇒ A linear relationship between the system state, $\boldsymbol{x}$, and the measurement vector, $\boldsymbol{z}$, in the form of (1) can now be established.
⇒ Using only synchronized phasor measurements, the measurement vector is given by the **complex measurements**

$$\boldsymbol{z} = \begin{bmatrix} \tilde{\boldsymbol{V}}^{measured} \\ \tilde{\boldsymbol{I}}^{measured} \end{bmatrix} \tag{8}$$

The linear function $\boldsymbol{h}(\boldsymbol{x})$ relating the state vector $\tilde{\boldsymbol{V}}$, and the complex currents and voltages is given by (7), rewriting

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{U} \\ \tilde{\boldsymbol{y}}\boldsymbol{A}^T + \tilde{\boldsymbol{y}}_{\boldsymbol{S}} \end{bmatrix} \tilde{\boldsymbol{V}} = \tilde{\boldsymbol{B}}\tilde{\boldsymbol{V}} \tag{9}$$

where $\boldsymbol{U}$ is an identity matrix with the rows corresponding to unmetered buses removed, and $\tilde{\boldsymbol{B}}$ is a complex matrix.
→The relationship between the states and measurements is

$$\begin{aligned} \boldsymbol{z} \quad &= \quad \boldsymbol{h}(\boldsymbol{x}) \quad + \quad \boldsymbol{e} \\ \begin{bmatrix} \tilde{\boldsymbol{V}}^{measured} \\ \tilde{\boldsymbol{I}}^{measured} \end{bmatrix} &= \quad \tilde{\boldsymbol{B}}\tilde{\boldsymbol{V}} \quad + \begin{bmatrix} \boldsymbol{e}_{\tilde{V}} \\ \boldsymbol{e}_{\tilde{I}} \end{bmatrix} \end{aligned} \tag{10}$$

where $\boldsymbol{e}_{\tilde{V}}$ and $\boldsymbol{e}_{\tilde{I}}$ are errors on the voltage and current phasors, respectively.

Phadke and Thorp's Linear State Estimator [4] - III

⇒ The solution to (10) is given by

$$\tilde{\boldsymbol{G}}\boldsymbol{x} = \tilde{\boldsymbol{B}}^{\dagger}\boldsymbol{R}\boldsymbol{z} \tag{11}$$

where the $\tilde{\boldsymbol{G}}$ is a complex gain matrix given by $\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{B}}^{\dagger}\boldsymbol{R}\tilde{\boldsymbol{B}}$ and $\boldsymbol{R}$ contains the covariances of the complex measurements, the measurements are assumed to be uncorrelated.($\dagger$: complex conjugate transpose of a matrix.)

⇒ The SE algorithm consists of computing the right hand side of (11) for each measurement scan and performing triangular factorization, then (11) is solved for $\boldsymbol{x}$ using back substitution

→ The solution is direct (finite number of operations), i.e. non-iterative, as opposed to the iterative solutions that Gauss-Newton procedures in Conventional SE

⇒Some comments about $\tilde{\boldsymbol{G}}$

→ If there are no network topology or measurement configuration changes between network scans then $\tilde{\boldsymbol{G}}$ is constant, the LU factors of $\tilde{\boldsymbol{G}}$ are computed once and used sequentially.

→ If the voltage phasor is measured at <u>all buses</u>, $\tilde{\boldsymbol{G}}$ is the covariance matrix of the measurements - real, and diagonal

→ When current phasors are measured at <u>both ends</u> of an element: $\tilde{\boldsymbol{G}}$ is symmetric and real.

→ When current phasors are measured at <u>one end</u> of an element: $\tilde{\boldsymbol{G}}$ is symmetric. The imaginary part of off-diagonal elements is small for normal X/R ratios.

Phadke and Thorp's Linear State Estimator [4] - IV

⇒Note that the state estimation solution (11) is obtained in rectangular coordinates. The results should be mapped into polar coordinates to be meaningful.
The state estimates $\tilde{\boldsymbol{V}}$ are given by[1]

$$\tilde{\boldsymbol{V}} = \tilde{\boldsymbol{G}}^{-1} \tilde{\boldsymbol{B}}^{\dagger} \boldsymbol{R} \begin{bmatrix} \tilde{\boldsymbol{V}}^{measured} \\ \tilde{\boldsymbol{I}}^{measured} \end{bmatrix} \tag{12}$$

which in polar coordinates are given by

$$\boldsymbol{V} = |\tilde{\boldsymbol{V}}|, \text{ and } \boldsymbol{\theta} = \angle \tilde{\boldsymbol{V}} \tag{13}$$

⇒ Covariances for the measurements - The standard deviations for bus voltage and line current measurements are given by [3, 4]

$$\sigma_{\tilde{\mathrm{V}}_i} = 0.0017 f_{\mathrm{SV}} + 0.005|\tilde{V}_i|; \;\; \sigma_{\tilde{\mathrm{I}}_i} = 0.0017 f_{\mathrm{SI}} + 0.01|\tilde{I}_i| \tag{14}$$

where f_{SV} and f_{SI} are the full scale value of the voltage and current measurement devices, respectively. Full scale values for f_{SV} are between 1-1.2 p.u. voltage. For f_{SI} they depend on the flow level in the network, a value of 5.0 p.u. for a 500 MVA flow in a 345 kV line is reported in [4]

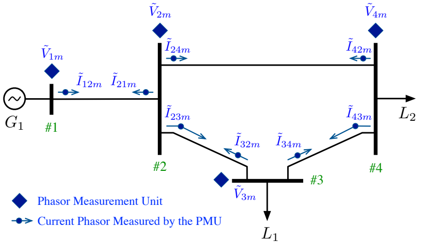
[1] $\tilde{\boldsymbol{G}}^{-1}$ is not computed, as discussed before.

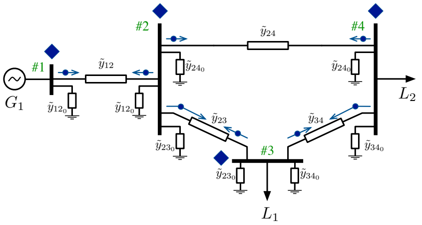
Summarizing the Procedure for Linear State Estimation

1. Obtain measurements and device status.
2. Based on the device status build the bus measurement admittance matrix $\tilde{\boldsymbol{Y}}$
   2.1. Obtain the current measurement bus incidence matrix $\boldsymbol{A}^T$
   2.2. Obtain the primitive matrix of series admittances of metered elements $\tilde{\boldsymbol{y}}$
   2.3. Obtain the shunt primitive matrix of metered shunt admittances $\tilde{\boldsymbol{y}}_{\boldsymbol{s}}$
   2.4. Compute $\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{y}}\boldsymbol{A}^T + \tilde{\boldsymbol{y}}_{\boldsymbol{s}}$
3. Obtain matrix $\tilde{\boldsymbol{B}}$
   3.1. Based on the metered voltage phasors obtain the $\boldsymbol{U}$
   3.2. Compute $\tilde{\boldsymbol{B}}$ using $\boldsymbol{U}$ and $\tilde{\boldsymbol{Y}}$
4. Based on the metered phasors
   4.1. Obtain the covariance matrix $\boldsymbol{R}$ using (14)
   4.2. Obtain the measurement vector $\boldsymbol{z}$
5. Solve $\tilde{\boldsymbol{G}}\boldsymbol{x} = \tilde{\boldsymbol{B}}^\dagger \boldsymbol{R}\boldsymbol{z}$
   5.1. Compute $\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{B}}^\dagger \boldsymbol{R}\tilde{\boldsymbol{B}}$
   5.2. Solve $\tilde{\boldsymbol{G}}\boldsymbol{x} = \tilde{\boldsymbol{B}}^\dagger \boldsymbol{R}\boldsymbol{z}$ by performing triangularization and back-substitution. Store the upper triangular matrix.

⇒ For a new snapshot of measurements – if there is no measurement configuration or network topology changes, only steps 4. and 5. need to be performed. Otherwise, return to step 1.

Example: All $\tilde{V}$ and $\tilde{I}$ are measured - I





Current Measurement Bus Incidence Matrix $\boldsymbol{A}^T$

$$
\boldsymbol{A}^T = \begin{array}{c} \tilde{I}_{12} \\ \tilde{I}_{21} \\ \tilde{I}_{23} \\ \tilde{I}_{32} \\ \tilde{I}_{34} \\ \tilde{I}_{43} \\ \tilde{I}_{24} \\ \tilde{I}_{42} \end{array}
\overset{\begin{array}{cccc} \text{B.1} & \text{B.2} & \text{B.3} & \text{B.4} \end{array}}{\begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & -1 & 1 \\
0 & 1 & 0 & -1 \\
0 & -1 & 0 & 1
\end{bmatrix}}
$$

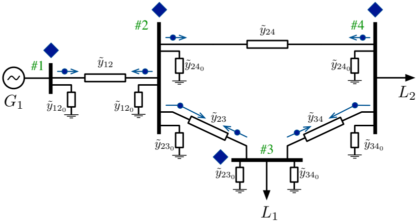
Primitive matrix of series admittances of metered elements $\tilde{\boldsymbol{y}}$

$$
\tilde{\boldsymbol{y}} = \text{diag}([\tilde{y}_{12}\ \tilde{y}_{21}\ \tilde{y}_{23}\ \tilde{y}_{32}\ \tilde{y}_{34}\ \tilde{y}_{43}\ \tilde{y}_{24}\ \tilde{y}_{42}])
$$

Note that $\tilde{y}_{12} = \tilde{y}_{21}$, $\tilde{y}_{23} = \tilde{y}_{32}$, $\tilde{y}_{34} = \tilde{y}_{43}$, and $\tilde{y}_{24} = \tilde{y}_{42}$.

Example: All $\tilde{V}$ and $\tilde{I}$ are measured - II

Shunt primitive matrix of shunt admittances metered ends $\tilde{\boldsymbol{y}}_{\boldsymbol{s}}$



$$\boldsymbol{y_s} = \begin{array}{c} \\ \tilde{I}_{12} \\ \tilde{I}_{21} \\ \tilde{I}_{23} \\ \tilde{I}_{32} \\ \tilde{I}_{34} \\ \tilde{I}_{43} \\ \tilde{I}_{24} \\ \tilde{I}_{42} \end{array} \begin{array}{c} \begin{array}{cccc} \text{B.1} & \text{B.2} & \text{B.3} & \text{B.4} \end{array} \\ \begin{bmatrix} \tilde{y}_{12_0} & 0 & 0 & 0 \\ 0 & \tilde{y}_{21_0} & 0 & 0 \\ 0 & \tilde{y}_{23_0} & 0 & 0 \\ 0 & 0 & \tilde{y}_{32_0} & 0 \\ 0 & 0 & \tilde{y}_{34_0} & 0 \\ 0 & 0 & 0 & \tilde{y}_{43_0} \\ 0 & \tilde{y}_{24_0} & 0 & 0 \\ 0 & 0 & 0 & \tilde{y}_{42_0} \end{bmatrix} \end{array}$$

Note that $\tilde{y}_{12_0} = \tilde{y}_{21_0}$, $\tilde{y}_{23_0} = \tilde{y}_{32_0}$, $\tilde{y}_{34_0} = \tilde{y}_{43_0}$, and $\tilde{y}_{24_0} = \tilde{y}_{42_0}$.

Bus measurement admittance matrix $\tilde{\boldsymbol{Y}}$

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{y}}\boldsymbol{A}^T + \tilde{\boldsymbol{y}}_{\boldsymbol{s}} = \begin{bmatrix} \tilde{y}_{12} + \tilde{y}_{12_0} & -\tilde{y}_{12} & 0 & 0 \\ -\tilde{y}_{12} & \tilde{y}_{12} + \tilde{y}_{21_0} & 0 & 0 \\ 0 & \tilde{y}_{23} + \tilde{y}_{23_0} & -\tilde{y}_{23} & 0 \\ 0 & -\tilde{y}_{23} & \tilde{y}_{23} + \tilde{y}_{23_0} & 0 \\ 0 & 0 & \tilde{y}_{34} + \tilde{y}_{34_0} & -\tilde{y}_{34} \\ 0 & 0 & -\tilde{y}_{34} & \tilde{y}_{34} + \tilde{y}_{34_0} \\ 0 & \tilde{y}_{24} + \tilde{y}_{24_0} & 0 & -\tilde{y}_{24} \\ 0 & -\tilde{y}_{24} & 0 & \tilde{y}_{24} + \tilde{y}_{24_0} \end{bmatrix}$$

Example: All $\tilde{V}$ and $\tilde{I}$ are measured - III

Constructing $\tilde{\boldsymbol{B}}$

⇒Line parameters (all in p.u.): L_{1-2}: r=0, x=0.01, b=0.001; L_{2-3}, L_{3-4}: r=0.01, x=0.1, b=0.15; L_{2-4}: r=0.02, x=0.2, b=0.3

⇒Primitive matrix of series admittances:

$$\tilde{\boldsymbol{y}} = \text{diag}([-j100,\ \ -j100,\ \ 0.9901 - j9.901,\ \ 0.9901 - j9.901,\ \ 0.9901 - j9.901,\\ 0.9901 - j9.901,\ \ 0.49505 - j4.9505,\ \ 0.49505 - j4.9505])$$

⇒Shunt primitive matrix:

$$\tilde{\boldsymbol{y}}_{\boldsymbol{s}}(1,1) = j0.0005,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(2,2) = j0.0005,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(3,2) = j0.075,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(4,3) = j0.075,\\ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(5,3) = j0.075,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(6,4) = j0.075,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(7,2) = j0.15,\ \ \tilde{\boldsymbol{y}}_{\boldsymbol{s}}(8,4) = j0.15,$$

⇒Bus measurement admittance matrix:

$$\tilde{\boldsymbol{Y}} = \begin{bmatrix} -j99.99 & j100 & 0 & 0 \\ j100 & -j99.99 & 0 & 0 \\ 0 & 0.99 - j9.826 & -0.99 + j9.9 & 0 \\ 0 & -0.99 + j9.9 & 0.99 - j9.826 & 0 \\ 0 & 0 & 0.99 - j9.826 & -0.99 + j9.9 \\ 0 & 0 & -0.99 + j9.9 & 0.99 - j9.826 \\ 0 & 0.495 - j4.8 & 0 & -0.495 + j4.95 \\ 0 & -0.495 + j4.95 & 0 & 0.495 - j4.8 \end{bmatrix}$$

⇒ $\tilde{V}_m$ phasors measured at all buses, thus: $\boldsymbol{U} = \text{diag}([1\ \ 1\ \ 1\ \ 1])$

Example: All $\tilde{V}$ and $\tilde{I}$ are measured - IV

The gain matrix $\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{B}}^{\dagger}\boldsymbol{R}\tilde{\boldsymbol{B}}$

⇒ For simplicity, $\sigma_{\tilde{V}_i} = 0.00187$ (f_{SV}=1.1); and $\sigma_{\tilde{I}_i} = 0.0085$ (f_{SI}=5).

⇒ $\boldsymbol{R} = \text{diag}([\sigma_{V_1}\sigma_{V_2}\sigma_{V_3}\sigma_{I_{12}}\ldots\sigma_{I_{24}}])$

⇒ All elements are measured from both ends, the gain matrix is real and symmetric

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} 1.445 & -1.445 & 0 & 0 \\ -1.445 & 1.4627 & -0.0142 & -0.0034694 \\ 0 & -0.0142 & 0.028404 & -0.0142 \\ 0 & -0.0034694 & -0.0142 & 0.017675 \end{bmatrix}$$

Linear SE Solution → solve $\tilde{\boldsymbol{G}}\boldsymbol{x} = \tilde{\boldsymbol{B}}^{\dagger}\boldsymbol{R}\boldsymbol{z}$

Simulation Settings

⇒ Measurements are simulated using the loadflow solution of the system

⇒ The resulting measurement vector $\boldsymbol{z}$ is

$$\begin{aligned}
\tilde{V}_{1m} &= 1.05 \angle 20 &&= 0.98668 + j0.35912 \\
\tilde{V}_{2m} &= 1.0272 \angle 17.5146 &&= 0.97956 + j0.30913 \\
\tilde{V}_{3m} &= 0.88654 \angle {-1.4645} &&= 0.88625 - j0.022658 \\
\tilde{V}_{4m} &= 0.87867 \angle {-1.4631} &&= 0.87839 - j0.022436 \\
\tilde{I}_{12m} &= 5.0493 \angle {-8.0923} &&= 4.999 - j0.71078 \\
\tilde{I}_{21m} &= 5.0498 \angle 171.8972 &&= -4.9994 + j0.71176 \\
\tilde{I}_{23m} &= 3.3946 \angle {-8.8449} &&= 3.3542 - j0.52195 \\
\tilde{I}_{32m} &= 3.44 \angle 168.9065 &&= -3.3757 + j0.66189 \\
\tilde{I}_{34m} &= 0.013693 \angle {-57.8668} &&= 0.0072833 - j0.011596 \\
\tilde{I}_{43m} &= 0.144 \angle 91.5525 &&= -0.0039013 + j0.14394 \\
\tilde{I}_{24m} &= 1.656 \angle {-6.5814} &&= 1.6451 - j0.18981 \\
\tilde{I}_{42m} &= 1.7519 \angle 164.4893 &&= -1.6881 + j0.4685
\end{aligned}$$

Note that $\boldsymbol{z}$ is complex, values in polar form are shown for reference.

Solution

⇒ Compute LU factors and solve by back substitution, in MATLAB use the backslash "\" operator

⇒ Voltage Magnitudes

Bus	$\lvert\tilde{V}^{\text{true}}\rvert = \lvert\tilde{V}^{\text{meas}}\rvert$	$\lvert\tilde{V}^{\text{se}}\rvert$
1	1.05	1.05
2	1.0272	1.0272
3	0.88654	0.88654
4	0.87867	0.87867

⇒ Voltage Angles

Bus	$\lvert\theta^{\text{true}}\rvert = \lvert\theta^{\text{meas}}\rvert$	$\lvert\theta^{\text{se}}\rvert$
1	20	20
2	17.5146	17.5146
3	-1.4645	-1.4645
4	-1.4631	-1.4631

⇒ No surprise that the measured and estimated values are the same as the true values from the loadflow were used as measurements.

Example: All $\tilde{V}$ and $\tilde{I}$ are measured - V

Adding Gaussian White Noise to the Measurements

Simulation Settings

⇒ Measurements are simulated using the same loadflow solution of the system

⇒ Realistically any measurement will have metering errors and noise.

⇒ For simplicity white gaussian noise is added to the load flow solution with an `snr` of 75 dB

⇒ The resulting measurement vector $\boldsymbol{z}$ is similar to the one of the previous example, but now it contains noise in the voltage and current phasors.

Solution

Voltage Magnitudes

Bus	$\|\tilde{V}^{\text{true}}\|$	$\|\tilde{V}^{\text{meas}}\|$	$\|\tilde{V}^{\text{se}}\|$	Residual
1	1.05	1.0506	1.046	0.0046402
2	1.0272	1.0277	1.0231	0.0045789
3	0.88654	0.88631	0.88229	0.004017
4	0.87867	0.87919	0.8748	0.0043947

Voltage Angles

Bus	$\|\theta^{\text{true}}\|$	$\|\theta^{\text{meas}}\|$	$\|\theta^{\text{se}}\|$	Residual
1	20	20.0017	20.017	0.015237
2	17.5146	17.5144	17.5194	0.0050314
3	-1.4645	-1.4628	-1.549	0.086146
4	-1.4631	-1.4636	-1.5505	0.086869

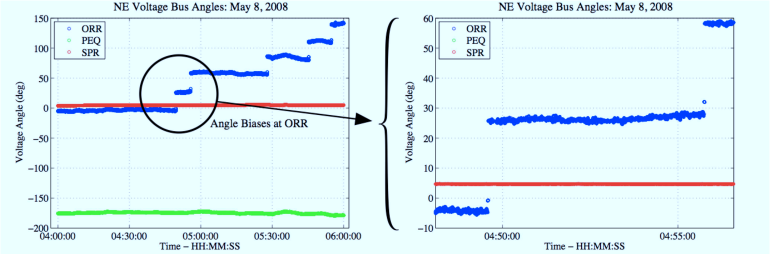
⇒ Residuals are acceptable for the `snr` used.

⇒ Additional Slides at the end of the presentation provide more examples.

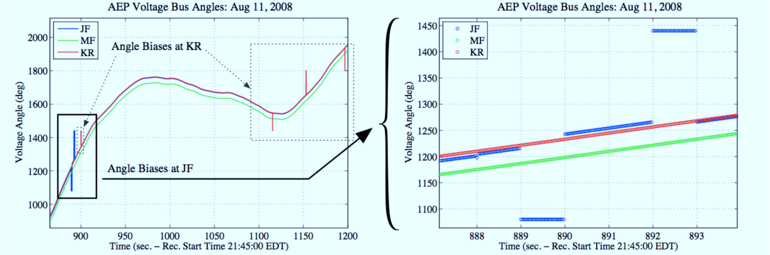
Angular Errors Observed in PMU Data

⇒ Voltage and current phasor angles exhibit persistent biases, random shifts, and other type of erros (shown latter)

Angle Biases in AEP



Angle Biases in NE



Characteristics of Angular Errors found in PMU Data

Phase Angle Errors are Attributed to

- A particular signal processing algorithm used by the PMU
- Errors with time synchronization (GPS signal overload), and internal clock synchronization with GPS
- Length of instrumentation cables
- PMU software and firmware
- Off-nominal operation

Why should we worry about them?

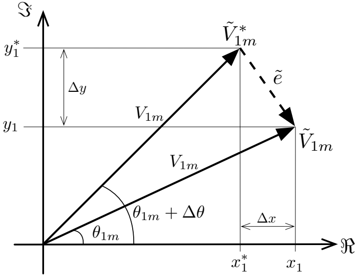
- **Uncorrected phasor data needs to be rejected as bad data if used in conventional SE!**
- It may cause difficulties in convergence otherwise.

Observed characteristic

$\Rightarrow$ When large angle shift occurs, the same bias appears in all voltage & current angles

- *The fact that the same angle shift will be present in all angles allows the PSE to correct for the shifts*

Angle Error Modeling in Polar Coordinates



Angle error in a measured phasor:

- $\tilde{V}_{1m} = V_{1m}\varepsilon^{j\theta_{1m}} \rightarrow$ is the measured voltage phasor
- $\tilde{V}_{1m}^* = V_{1m}\varepsilon^{\theta_{1m}+\theta_e}$ is the phasor measurement $\tilde{V}_1$ with an angle error of θ_e,
- $\tilde{e} = \tilde{V}_1 - \tilde{V}_1^*$ is the error phasor.

$\Rightarrow$ *Angle error can be modeled in* <u>*polar coordinates*</u> *with a linear angular unknown* $\phi = \theta_e$ *wich is not reliant on the phasor magnitude* V_{1m}.

$\Rightarrow$ Rectangular coord. would need to account for Δx and Δy in $\tilde{V}_{1m}^*$.

$\Rightarrow$ Δx and Δy are dependent of V_{1m} and a nonlinear function of θ_{1m} and θ_e.

$\Rightarrow$ Rectangular coord. model allows a non-iterative solution, **but**, *does't allow to model the phase and magnitude errors independently*

$\Rightarrow$ Polar coord. is more appropriate $\rightarrow$ V and θ are mostly uncorrelated variables as obtained by PMUs.

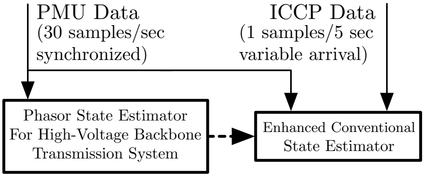
Phasor State Estimation [5, 6, 7]

L. Vanfretti, J.H. Chow, S. Sarawgi, and B. Fardanesh, "A Phasor-Data Based State Estimator Incorporating Phase Bias Correction," to appear, IEEE Transactions on Power Systems. Accepted 03/2009.

⇒ A new approach for SE *based only on PMUs*

⇒ Requires a *modest* number of PMUs installed in HV substations

⇒ It can supplement a conventional SE based on ICCP and PMU data



Why is this approach attractive?

⇒ Allows for a PMU-based SE implementation without disrupting the available state estimator

⇒ *Standalone Estimator* - provides visibility of the HV network even when the SE is out, adding reliability

⇒ *Problem Formulation* - provides for a *special kind of bad data detection*

Measurement Model in Polar Coordinates

Model Buses and Lines

⇒ Starts from the buses with PMUs and enables estimation of portions with connectivity to PMU buses.

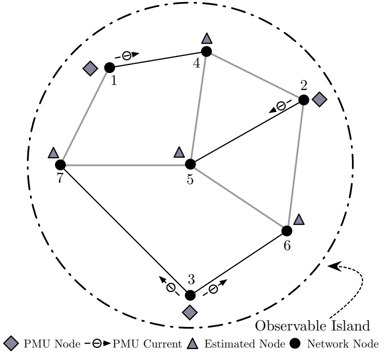
⇒ Having determined the observable islands, the state vector is formed by

- All measured voltages **and currents** (both, $\|\cdot\|$ and $\angle\cdot$)
- *All phasor-estimable voltages and currents* (both, $\|\cdot\|$ and $\angle\cdot$)

⇒ In a system with N buses: N_1 buses with PMUs have measured voltage phasors and N_2 non-PMU buses, $N = N_1 + N_2$

⇒ There are L lines: L_1 lines with PMUs have measured current phasors and L_2 are unmonitored, $L = L_1 + L_2$

⇒ Example:
$N = 7$, $N_1 = 3$ (1, 2, 3), $N_2 = 4$ (4,5,6,7)
$L = 10$, $L_1 = 4$ (1-4, 2-5, 3-6, 3-7), $L_2 = 6$ (1-7, 2-4, 2-6, 4-5, 5-6, 5-7)



State Vector:

$$x = \begin{bmatrix} V & I & \theta & \delta \end{bmatrix}^T$$

$$V = \begin{bmatrix} V_1 & \cdots & V_{N_1} & V_{N_1+1} & \cdots & V_N \end{bmatrix}^T$$

$$\theta = \begin{bmatrix} \theta_1 & \cdots & \theta_{N_1} & \theta_{N_1+1} & \cdots & \theta_N \end{bmatrix}^T$$

$$I = \begin{bmatrix} \cdots & I_{ik} & \cdots \end{bmatrix}^T$$

$$\delta = \begin{bmatrix} \cdots & \delta_{ik} & \cdots \end{bmatrix}^T$$

Measurement and Network Model

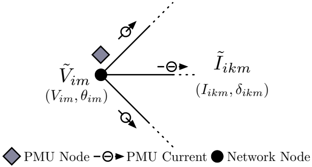
Measurement Model

⇒ At each PMU Bus the available measurements are V_{im}, θ_{im}, I_{ikm}, and δ_{ikm}, we pair each voltage measurement to it's respective state

$$V_i = V_{im} + e_{V_i}, \quad \theta_i = \theta_{im} + e_{\theta_i} \quad (15)$$

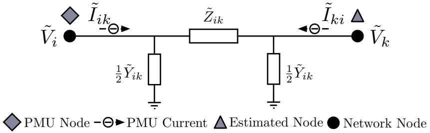
⇒ Current Measurement Equations

$$I_{ik} = I_{i_k m} + e_{I_{ik}},\ \delta_{ik} = \delta_{ikm} + e_{\delta_{ik}} \quad (16)$$



⇒ e_{V_i} and e_{θ_i} are voltage mag. and angle measurement errors, and $e_{I_{ik}}$ and $e_{\delta_{ik}}$ are current mag. and angle measurement errors

Network Model



⇒ Based on circuit equations using the equivalent circuit

$$\tilde{V}_k = \tilde{V}_i - \tilde{Z}_{ik}\left(\tilde{I}_{ik} - \tfrac{1}{2}\tilde{Y}_{ik}\tilde{V}_i\right) \quad (17)$$

⇒ Each equation is complex → divided into $\Re$ & $\Im$ parts

$$f_{ik} = \left(1 + \tfrac{1}{2}\tilde{Y}_{ik}\tilde{Z}_{ik}\right)\tilde{V}_i - \tilde{Z}_{ik}\tilde{I}_{ik} - \tilde{V}_k \quad (18)$$

$$f_{ik\text{re}} = \text{Re}(f_{ik}) = 0, \quad f_{ik\text{im}} = \text{Im}(f_{ik}) = 0 \quad (19)$$

⇒ And assembled into an $2L$ dimensional nonlinear vector f

$$f = \begin{bmatrix} \cdots & f_{ik\text{re}} & f_{ik\text{im}} & \cdots \end{bmatrix}^T \quad (20)$$

PSE Solution - WLS Formulation

⇒ **Satisfy** the network equations **while minimizing** the measurement errors in the measurement equations

$$\min_x q(x), \quad \text{subject to}: \quad f = 0 \tag{21}$$

$$q(x) = \frac{1}{2}\left(\|W_V e_V\|^2 + \|W_I e_I\|^2 + \|W_\theta e_\theta\|^2 + \|W_\delta e_\delta\|^2\right) \tag{22}$$

⇒ f is a nonlinear function of V, I, θ, and δ → augment equality constraint $f = 0$ to the objective function $q(x)$

$$q'(x) = q(x) + \frac{1}{2}\|W_f f\|^2 = \frac{1}{2}\|W h(x)\|^2 \tag{23}$$

and W_f is a diagonal matrix with unity weights,

$$h = \begin{bmatrix} f^T & e^T \end{bmatrix}^T, \quad e = \begin{bmatrix} e_V^T & e_I^T & e_\theta^T & e_\delta^T \end{bmatrix}^T \tag{24}$$

$$W = \text{block} - \text{diag}\begin{pmatrix} W_f & W_V & W_I & W_\theta & W_\delta \end{pmatrix} \tag{25}$$

⇒Constrained WLS problem (21) transformed into an unconstrained WLS problem

$$\min_x q'(x) \tag{26}$$

PSE Solution - Successive Solution Algorithm

PSE Solution

⇒ When $L_1 = N_2$ (no. of constrained eqns. = to no. of unknowns) → ∃ a unique solution (Measurement errors are taken to be zero)

⇒ When $L_1 > N_2$ (no. of constrained eqns. > to no. of unknowns) → ∃ a WLS best fit

Weighting Matrices

⇒ Diagonal matrices, calculated depending on the type of measurement eqn. - most weights are unity except for weights on current magnitudes (lower for heavily loaded lines)

$$W_I = \text{diag}\left(\cdots, \min\left(1, \frac{1}{I_{ikm}}\right), \cdots\right) \tag{27}$$

Iterative Solution

⇒ WLS is *solved successively* using the Newton-type methods

⇒ In the Gauss-Newton method the increment Δx is computed as

$$\Delta x = -(H(x_c))^{-1}(WJ(x_c))^T h(x_c) \tag{28}$$

The new solution is updated to $x_c + \Delta x$ and the Gauss-Newton iteration (28) is repeated until convergence.

Jacobian Structure

$$J = \begin{bmatrix} & \partial f/\partial x & & \\ \hline U_V & 0 & 0 & 0 \\ \hline 0 & 0 & U_I & 0 \\ \hline 0 & U_\theta & 0 & 0 \\ \hline 0 & 0 & 0 & U_\delta \end{bmatrix}$$

where U is an identity matrix → $\partial e/\partial x$

Observability

⇒ PSE requires that the Jacobian has full rank and is equal to the number of unknowns

$$\text{rank}(J) = U_T = 2(N + L) \tag{29}$$

⇒This rank condition is satisfied when the network is observable.

⇒ If every bus in the PSE is connected to all other buses → ∃ a single PSE island

⇒ Otherwise, a topological algorithm isolates the islands → makes all possible $\tilde{\boldsymbol{V}}$ and $\tilde{\boldsymbol{I}}$ observable

⇒ A PSE model is constructed for each island → N voltages and L currents will be observable

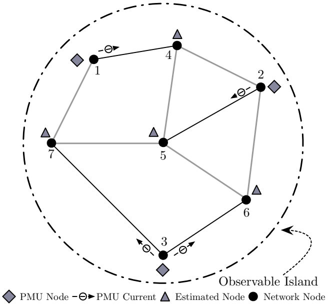
- Unknowns: $U_T = 2(N + L)$
- Model Eqns.: $E_T = 2(L + N_1 + L_1)$

⇒ Hence, for an obs. PSE network we need

$$N_1 + L_1 \geq N \tag{30}$$

⇒ With relay-based PMUs, (30) reduces to $2L_1 \geq N$, that is

$$L_1 \geq \text{int}[N/2] \tag{31}$$



- Current Measurements: $L_1 = 4$
- Non-PMU Buses: $N_2 = 4$
- $L_1 = N_2$, holds as an equality.

Extension for Angle Correction

Measurement Model

⇒The measurement equations can be updated to

$$\theta_i = \theta_{im} - \phi_i + e_{\theta_i} \tag{32}$$

$$\delta_{ik} = \delta_{ikm} - \phi_i + e_{\delta_{ik}} \tag{33}$$

Bus 1 is the island reference → angle bias will not be applied to it.
⇒ The angle bias terms form a vector

$$\phi = \begin{bmatrix} \phi_2 & \phi_3 & \cdots & \phi_{N_1} \end{bmatrix}^T \tag{34}$$

⇒ Thus the WLS problem (26) can be modified to the PSE-Φ problem of

$$\min_{x_\phi} q'(x_\phi) \tag{35}$$

where

$$x_\phi = \begin{bmatrix} x^T & \phi^T \end{bmatrix}^T \tag{36}$$

and $e(x_\phi)$ has been modified to incorporate (32) and (33).

Extended Jacobian

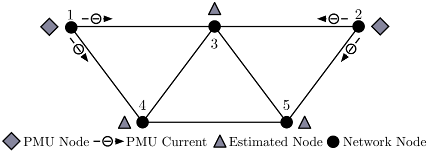
⇒The Jacobian matrix is expanded to

$$J_\phi = \left[\begin{array}{c|c} J & \begin{array}{c} 0 \\ \hline \partial e(\phi)/\partial\phi \end{array} \end{array} \right] \tag{37}$$

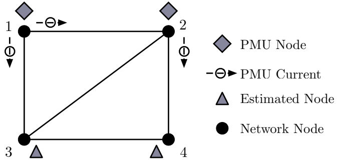
Note that $\partial e(\phi)/\partial\phi$ is sparse and consists of ones and zeros.

Redundancy

⇒ To perform angle bias correction *redundancy* is required → $L_1 > N_2$



- $\tilde{V}_3$ can be computed using the phasor measurements from Bus 1 or Bus 2.
- If one of the phasor measurements has an angle bias, the two computed values of $\tilde{V}_3$ will be different.
- Correcting the angle bias allows for all 5 voltages to be accurately computed.



- $\tilde{V}_1$ can be used to compute $\tilde{V}_2$, providing redundancy.
- Voltage phasor measurements at adjacent buses ($\tilde{V}_1$ & $\tilde{V}_2$) can be used to check for the phasor current.

- To achieve redundancy for angle bias correction, the no. of measured *current* phasors of

$$L_1 \geq N - 1 \tag{38}$$

  are required along with the corresponding voltage phasors.
- This inequality is derived from a Jacobian rank condition.

Redundancy Rank Condition

⇒ To correct angle shifts it is necessary to have redundant measurementst, thus the rank condition on the extended Jacobian is updated to

$$\text{rank}(J_\phi) = U_{T\phi} \tag{39}$$

where $U_{T\phi}$ is the total number of unknown variables which when including the angle biases, is $U_{T\phi} = 2(N + L) + (N_1 - 1)$.

⇒ Condition (39) can be used to determine the lower bound on the no. of measurements needed for redundancy

- No. of unknowns in $U_{T\phi}$ corresponding to angle unknowns is: $(N + L) + (N_1 - 1)$
- No. of rows in J_ϕ corresponding to angle unknowns is: $(L + N_1 + L_1)$
- Hence, to satisfy condition (39) it is required that the number of rows $(N + L) + (N_1 - 1)$ is greater than or equal to the number of unknowns $(N + L_1 + N_1)$

$$L + N_1 + L_1 \geq (N + L) + (N_1 - 1) \tag{40}$$

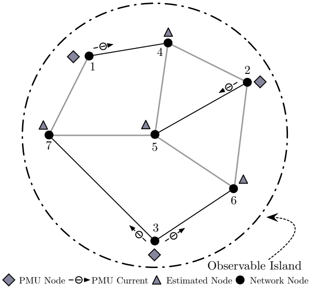
Which reduces to

$$L_1 \geq N - 1 \tag{41}$$

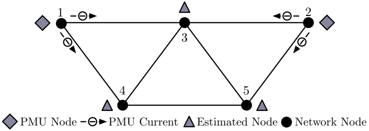
Note that $N_1 \geq 2$ as redundancy requires at least two voltage phasor measurements, and L_1 is an integer.

Example - Observability and Redundancy Analysis

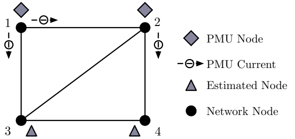
Network 1



Network 2



Network 3



Observability Analysis

Network	E_T	U_T	rank(J)	Observable
1	34	34	34	Yes
2	26	24	24	Yes
3	20	18	18	Yes

Redundancy Analysis

Network	Bias Terms	E_T	$U_{T\phi}$	rank(J_ϕ)	Redundant
1	$\phi_2,\ \phi_3$	34	36	34	No
2	ϕ_2	26	25	25	Yes
3	ϕ_2	20	19	19	Yes

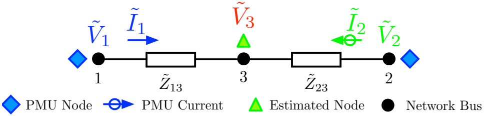
Summarizing the Procedure for Phasor State Estimation

1. Obtain measurements and device status.
2. Determine the observable islands, and the islands with enough redundancy for angle-bias correction

   For each island —

3. Based on the metered phasors, build the measurement model e, network model f, and state vector x.
   - 3.1. From these obtain the non-linear function $h(x)$ or $h(x_\phi)$
   - 3.2. Include phase-bias variables in x_ϕ if enough redundancy is available.
4. Obtain the Jacobian J or $J(\phi)$
5. Obtain the weighting matrix W
6. Obtain the SE solution using Gauss-Newton method.

Example: $\tilde{V}$ and $\tilde{I}$ measurements at Bus 1 and 2 - I



Measurements – an angle shift of 7.5° is present in the phasors at Bus 2

	$\lvert V_1 \rvert$	$\lvert V_2 \rvert$	θ_1	θ_2
True	1.05	1.05	20	20
Meas.	1	1	10	17.5

	$\lvert I_1 \rvert$	$\lvert I_2 \rvert$	δ_1	δ_2
True	5.7794	5.7794	37.9708	37.9708
Meas.	5.7794	5.7794	37.9708	45.4708

⇒ Circuit eqns.: $\tilde{V}_3 = \tilde{V}_1 - \tilde{Z}_1\tilde{I}_1, \quad \tilde{V}_3 = \tilde{V}_2 + \tilde{Z}_2\tilde{I}_2$

⇒ Network Model:

$$f_1 : \quad 0 = V_1 \cos\theta_1 - Z_1 I_1 \cos(\delta_1 + \alpha_1) - V_3 \cos\theta_3$$
$$f_2 : \quad 0 = V_1 \sin\theta_1 - Z_1 I_1 \sin(\delta_1 + \alpha_1) - V_3 \sin\theta_3$$
$$f_3 : \quad 0 = V_2 \cos\theta_2 + Z_2 I_2 \cos(\delta_2 + \alpha_2) - V_3 \cos\theta_3$$
$$f_4 : \quad 0 = V_2 \sin\theta_2 + Z_2 I_2 \sin(\delta_2 + \alpha_2) - V_3 \sin\theta_3$$

⇒ Measurement Model:

$$\left.\begin{aligned} e_1 &: e_{V_1} = V_1 - V_{1m} \\ e_2 &: e_{V_2} = V_2 - V_{2m} \\ e_3 &: e_{I_1} = I_1 - I_{1m} \\ e_4 &: e_{I_2} = I_2 - I_{2m} \end{aligned}\right\} \text{Magnitudes,} \quad \left.\begin{aligned} e_5 &: e_{\theta_1} = \theta_1 - \theta_{1m} \\ e_6 &: e_{\theta_2} = \theta_2 - \theta_{2m} + \phi \\ e_7 &: e_{\delta_1} = \delta_1 - \delta_{1m} \\ e_8 &: e_{\delta_2} = \delta_2 - \delta_{2m} + \phi \end{aligned}\right\} \text{Angles}$$

⇒ State Vector:

$$\boldsymbol{x} = [V_1 \ \ V_2 \ \ I_1 \ \ I_2 \ \ V_3 \ \ \theta_1 \ \ \theta_2 \ \ \delta_1 \ \ \delta_2 \ \ \theta_3 \ \ \phi]^T$$

Example: $\tilde{V}$ and $\tilde{I}$ measurements at Bus 1 and 2 - II

$\Rightarrow$ Jacobian Matrix

$\rightarrow$ Non-zero elements corresponding to f_1

$$\frac{\partial f_1}{\partial V_1} = \cos\theta_1, \quad \frac{\partial f_1}{\partial I_1} = -Z_1\cos(\delta_1 + \phi_1), \quad \frac{\partial f_1}{\partial V3} = -\cos\theta_3,$$
$$\frac{\partial f_1}{\partial \theta_1} = -V_1\sin\theta_1, \quad \frac{\partial f_1}{\partial \delta_1} = Z_1 I_1\sin(\delta_1 + \alpha_1), \quad \frac{\partial f_1}{\partial \theta_3} = V_3\sin\theta_3$$

$\rightarrow$ Non-zero elements corresponding to the measurement model

Magnitudes

$$\frac{\partial e_1}{\partial V_1} = 1, \quad \frac{\partial e_2}{\partial V_2} = 1$$
$$\frac{\partial e_3}{\partial I_1} = 1 \quad \frac{\partial e_4}{\partial I_2} = 1$$

Angles

$$\frac{\partial e_5}{\partial \theta_1} = 1, \quad \frac{\partial e_6}{\partial \theta_2} = 1, \quad \frac{\partial e_6}{\partial \phi} = 1$$
$$\frac{\partial e_7}{\partial \delta_1} = 1, \quad \frac{\partial e_8}{\partial \delta_2} = 1, \quad \frac{\partial e_8}{\partial \phi} = 1$$

$\Rightarrow$ The jacobian matrix J_ϕ, the weighting matrices W, and the nonlinear function $h(x_\phi)$ are computed at each iteration of the Gauss-Newton method – only show the first iteration here

$\Rightarrow$ Jacobian Matrix for the first iteration is

$\mathbf{J}_\phi =$

0.93969	0	-0.00016174	0	-0.94624	-0.35912	0	0.092736	0	0.33028	0
0.34202	0	-0.016046	0	-0.32346	0.98668	0	-0.00093475	0	-0.96621	0
0	0.95372	0	-0.0019341	-0.94624	0	-0.30071	0	-0.092065	0.33028	0
0	0.30071	0	0.01593	-0.32346	0	0.95372	0	-0.011178	-0.96621	0
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1

Example: $\tilde{V}$ and $\tilde{I}$ measurements at Bus 1 and 2 - III

⇒ Weighting matrices for the fist iteration – $W_V = \mathbf{diag}(\ldots\ \mathbf{min}(1, 1/V_i)\ \ldots)$, etc.

$$W_f = \mathbf{diag}([1\ \ 1\ \ 1\ \ 1]),$$

$$W_V = \mathbf{diag}([1\ \ 0.95238]),\quad W_I = \mathbf{diag}([0.17303\ \ 0.17303]),\quad W_\theta = \mathbf{diag}([1\ \ 1]),\quad W_\delta = \mathbf{diag}([1\ \ 1])$$

⇒ The nonlinear function (24) for the first iteration is

$$h(x_0) = [6.2075\times10^{-5}\ \ -1.7903\times10^{-5}\ \ -0.0042767\ \ -0.0011146\ \ 0.0023354\ \ -0.002112\ \ \ldots$$
$$-0.00044773\ \ -0.00030754\ \ 0.00010186\ \ -0.00018932\ \ 0.00018075\ \ 0.00018932]^T$$

⇒ With J_ϕ, $h(x_0)$, and W the increment Δx in (28) for the first iteration is

$$\Delta x = [0.0023354\ \ -0.002112\ \ -0.00044773\ \ -0.00030754\ \ 0.0022807\ \ 0.00010186\ \ \ldots$$
$$-0.13072\ \ 0.00018075\ \ -0.13034\ \ -0.065116\ \ 0.13053]^T$$

⇒ After two iterations convergence is reached for a tolerance of $\epsilon = 1\times10^{-12}$, and the solution obtained is

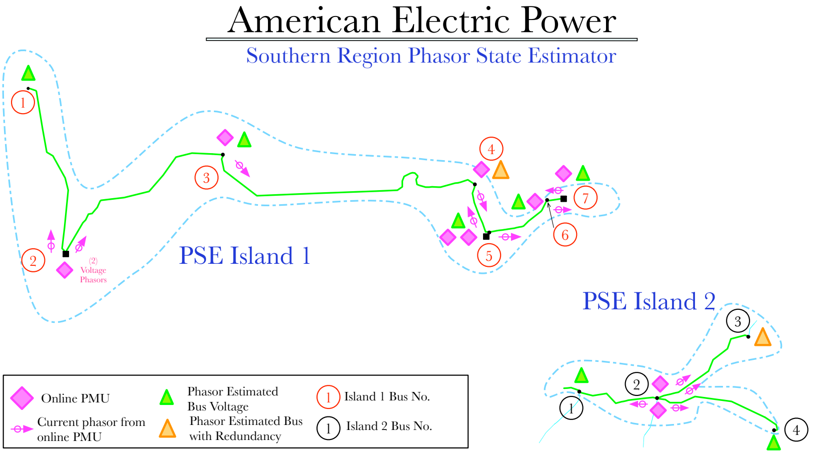
Magnitudes

	True	Meas.	Est.
V_1	1.05	1.05	1.05
V_2	1	1	1
I_1	5.7794	5.7794	5.7794
I_2	5.7794	5.7794	5.7794
V_3	1.0211	—	1.0211

Angles

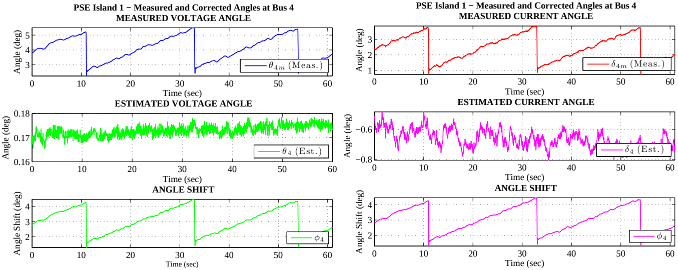
	True	Meas.	Est.
θ_1	20	20	20
θ_2	10	17.5	10
δ_1	37.9708	37.9708	37.9708
δ_2	37.9708	45.4708	37.9708
θ_3	15.122	—	15.122
ϕ	—	—	7.5

Application to AEPs HV Network

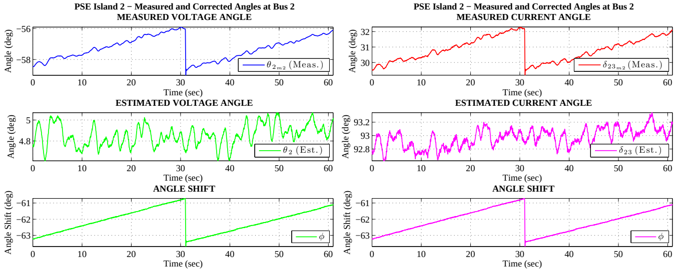


State Estimation and Phase Angle Error Correction

Island 1

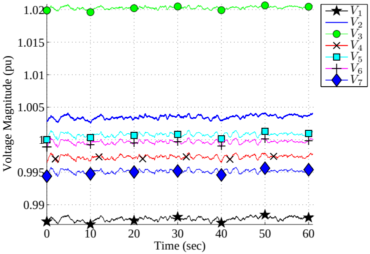


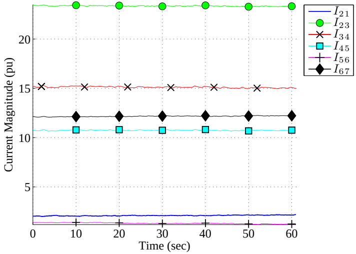
Island 2



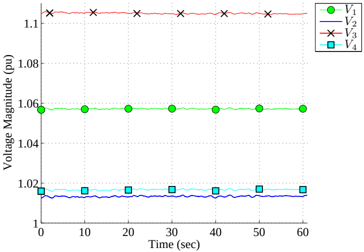
Voltage Magnitude and Current Magnitude Estimates

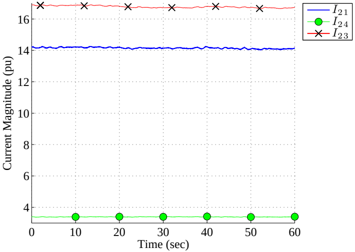
Island 1





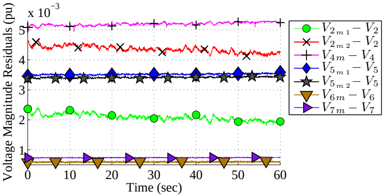
Island 2

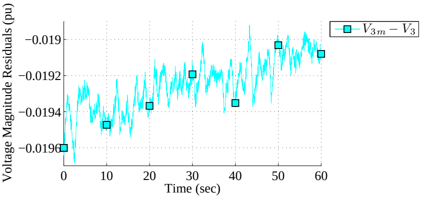




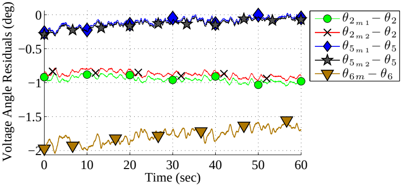
Voltage Phasor Measurement Residuals in Island 1

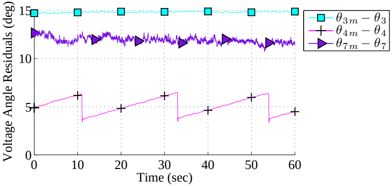
Voltage Magnitude Residuals



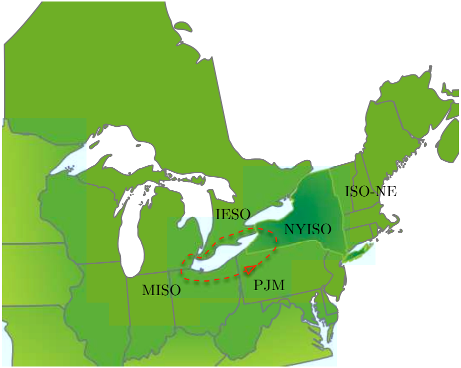


Voltage Angle Residuals





Using the PSE to Monitor the Erie Loop



- High level visibility of the sorrunding areas of NYS - based solely in PMU Data and HV network
- Potentially enable better monitoring and control of the Erie Loop Flow
- An independent state estimator from the currently available SEs
  - Will not suffer from difficulties caused by external network model inacurracies
  - Could aid in providing information for better external network modeling for conventional SEs

Conclusions

What did this presentation cover?

- Discussed the different approaches in incorporating PMU data onto Conventional SE
- Discussed two different approaches for SE with PMUs only: The Linear Phadke and Thorp SE, and the Phasor State Estimator
- Discussed about an specific type of error measurement - phase angle shifts or bias, which have been observed from archived records of PMUs installed in the field.
- The Linear SE has the advantage of relying on the solution of a *linear system of eqns.*, however it is not clear how to deal with phase angle shifts within these framework.
- The PSE concept has been developed and illustrated
  ⋄ The formulation extends for automatic detection and correction of angle biases that may exist in the PMU data
- The notion of PMU data redundancy to remove angle biases was presented - this is a new concept of redundancy different from those used in conventional SE
  ⋄ Provided observability and redundancy conditions in terms of the rank of a Jacobian matrix.
- Determined the minimum number of line current phasors required for redundancy.

Some other things to look at

- PMU Placement for SE – a topic worthy of a tutorial presentation for itself.
- Some interesting questions to answer
  ⋄ Staged placement - in what order should utilities place PMUs to maximize observability and minimize cost?
  ⋄ Securing Observability - where should utilities install PMUs so they do not lose observability given measurement loss or contingencies? (Some work has been done, but not everything has been said)
  ⋄ Redundancy for Angle-bias Correction

Thank you!

Questions?

References I

- A. Abur and A. G. Expósito, *Power System State Estimation: Theory and Implementation*. CRC Press, 2004.
- C. Kelley, *Solving Nonlinear Equations with Newton's Method*. Philadelphia, PA: SIAM, 2003.
- J. Allemong, L. Radu, and A. Sasson, "A fast and reliable state estimation algorithm for aep's new control center," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, no. 4, pp. 933–944, April 1982.
- A. G. Phadke, J. S. Thorp, and K. J. Karimi, "State Estimation with Phasor Measurements," *IEEE Transactions on Power Systems*, vol. 1, no. 1, pp. 233–238, Feb. 1986.
- L. Vanfretti, J. H. Chow, S. Sarawgi, D. Ellis, and B. Fardanesh, "A Framework for Estimation of Power Systems Based on Synchronized Phasor Measurement Data," *IEEE PES General Meeting*, July 2009.
- L. Vanfretti, "Phasor measurement-based state estimation of electric power systems and linearized analysis of power system network oscillations," Ph.D Thesis, Rensselaer Polytechnic Institute, Troy, NY, December 2009.
- L. Vanfretti, J. Chow, S. Sarawgi, and B. Fardanesh, "A Phasor Data-Based State Estimator Incorporating Phase Bias Correction," *IEEE Transactions on Power Systems*, to appear. Accepted, March 2010.
- J. Thorp, A. Phadke, and K. Karimi, "Real Time Voltage-Phasor Measurement For Static State Estimation," *IEEE Transactions on Power Systems*, vol. PAS-104, no. 11, pp. 3098–3106, Nov. 1985.

References II

- I. Slutsker, S. Mokhtari, L. Jaques, J. Provost, M. Perez, J. Sierra, F. Gonzalez, and J. Figueroa, “Implementation of phasor measurements in state estimator at Sevillana de Electricidad,” in *IEEE Power Industry Computer Application Conference Proceedings*, May 1995, pp. 392–398.
- M. Parashar, *et al*, “Imlementation of Phasor Measurements in SDG&E State Estimator,” California Energy Commission, PIER Program Energy Commission - 500-02-04 MR053, Tech. Rep., 2008.
- B. Fardanesh, “Use of Phasor Measurement in a Commercial (or Industrial) State Estimator,” EPRI, Palo Alto, California, Final Report 1011002, July 2004.
- R. Avila-Rosales, M. J. Rice, J. Giri, L. Beard, and F. Galvan, “Recent Experience with a Hybrid SCADA/PMU On-line State Estimator,” *IEEE Power and Energy Society General Meeting, Calgary*, July 2009.
- D. Atanackovic, J. Clapauch, G. Dwernychuk, J. Gurney, and H. Lee, “First steps to wide area control,” *IEEE Power and Energy Magazine*, vol. 6, no. 1, pp. 61–68, January-February 2008.
- L. Kondragunta, “Enahancement of State Estimation Results using Phasor Measurements,” in *North American SynchroPhasor Initiative (NASPI) Work Group Meeting, March 6-7, 2008*, New Orleans, LA, available online: http://www.naspi.org/meetings/workgroup/workgroup.stm.
- A. Ghassemian and B. Fardanesh, “Phasor Assisted State Estimation for NYS Transmission System - Implementation & Testing,” *IEEE Power System Conference and Exposition, Seattle, WA*, March 2009.
- R. Avila-Rosales, M. J. Rice, R. Lopez, L. Beard, T. Mathur, F. Galvan, V. Gupta, J. Lambert, J. Graffy, and M. Papic, “Impact of PMU Technology in State Estimation,” *Cigre 2008 Session Proceedings*, 2008.

Additional Reference Material

And Examples

Phasor Assisted State Estimation

Phasor Assisted State Estimation - I

Inclusion of Phasor Voltage *Angle* Measurements [8]

J. Thorp, A. Phadke, and K. Karimi, "Real Time Voltage-Phasor Measurement For Static State Estimation," IEEE Transactions on Power Systems, vol. PAS-104, no. 11, pp.3098-3106, Nov. 1985.

⇒ Incorporate the $\boldsymbol{\theta}_m$ calculated by PMUs into the measurement vector $\boldsymbol{z}$ (1)

⇒ *Assumption:* by measurement synchronization all angles are measured w.r.t a common reference, implying direct angle measurements.

⇒ The new measurement vector $\boldsymbol{z}$ organized in terms of active and reactive partitions is

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{z_A} \\ \boldsymbol{z_R} \end{bmatrix} \tag{42}$$

where

$$\boldsymbol{z_A} = \begin{bmatrix} \boldsymbol{P}_{ij} \\ \boldsymbol{P}_i \\ \boldsymbol{\theta}_i \end{bmatrix} \quad \begin{matrix} \text{active power line flow meas.} \\ \text{active power injection meas.} \\ \text{PMU bus angle meas.} \end{matrix}$$

$$\boldsymbol{z_R} = \begin{bmatrix} \boldsymbol{Q}_{ij} \\ \boldsymbol{Q}_i \\ \boldsymbol{V}_i \end{bmatrix} \quad \begin{matrix} \text{reactive power line flow meas.} \\ \text{reactive power injection meas.} \\ \text{voltage magnitude meas.} \end{matrix} \tag{43}$$

where $i = 1, \ldots, N$, N being the total number of buses in the network.

⇒ Appropiate modifications to $\boldsymbol{h}(\boldsymbol{x})$, Jacobian, and $\boldsymbol{R}$ are also required.

Phasor Assisted State Estimation - II

⇒ $\boldsymbol{h}(\boldsymbol{x})$ is augmented to include the relationship

$$f_{\theta_i} : \theta_i^{measured} = \theta_i + e_{\theta_i}$$

⇒ $\boldsymbol{H}(\boldsymbol{x})$ should be modified to include new rows for **phasor angle meas.**, the non-zero entries are

$$\frac{\partial f_{\theta_i}}{\partial \theta_i} = 1$$

⇒ Reference bus: choose a reference bus among the buses with PMUs, or measure the reference bus normally used by the conventional SE. Phase angle measurements are included into the SE as phase angle differences, i.e.

$$\theta_i = \theta_i^{measured} - \theta^{ref}$$

⇒ The first implementation is reported in:

I. Slutsker, S. Mokhtari, L. Jaques, J. Provost, M. Perez, J. Sierra, F. Gonzalez, and J. Figueroa, "Implementation of phasor measurements in state estimator at Sevillana de Electricidad," in IEEE Power Industry Computer Application Conference Proceedings, May 1995, pp. 392-398.

⇒ In a recent implementation by SDGE a PMU-bus reference was avoided by introducing the angle differences between PMUs as state variables instead of the measured bus angles

$$f_{\theta_i} : \Delta\theta^{measured} = \theta_i - \theta_j + e_{\Delta\theta}$$

with appropriate modifications to $\boldsymbol{R}$ and $\boldsymbol{H}(\boldsymbol{x})$.

Phasor Assisted State Estimation - III

Inclusion of Voltage and Current Synchrophasors

⇒ Complete voltage phasors (phase angle and magnitude) and current phasors included in $\boldsymbol{z}$
⇒ Jacobian, $\boldsymbol{H}(\boldsymbol{x})$, is augmented to include the sensitivities to the bus voltage angles θ_i (as above), and the complex currents $\tilde{I}_{ij} = I_{ij}\angle\delta_{ij}$. The complex current sensitivities are

$$\frac{\partial I_{ij}}{\partial V_i}, \quad \frac{\partial I_{ij}}{\partial V_j}, \quad \frac{\partial I_{ij}}{\partial \theta_i}, \quad \frac{\partial I_{ij}}{\partial \theta_j}, \quad \frac{\partial \delta_{ij}}{\partial V_i}, \quad \frac{\partial \delta_{ij}}{\partial V_j}, \quad \frac{\partial \delta_{ij}}{\partial \theta_i}, \quad \frac{\partial \delta_{ij}}{\partial \theta_j} \tag{44}$$

where I_{ij} and δ_{ij} is a PMU-measured current between buses i and j.
⇒ Implemented at NYPA.

Power Conversion Approach

⇒ Calculate the complex power $\boldsymbol{S}_{ij}$ (MW, and MVAr line flows) from the PMUs at each branch.
⇒ *Con.:* neither of the PMU-measured states ($\boldsymbol{V}$ and $\boldsymbol{\Theta}$) are used in the measurement vector.
Derived power flows are equivalent adding paired analog measurements.
⇒ *Pro.:* No modifications to SE formulation or software.
⇒ The addition of a PMU with multiple line current meas. is equivalent to adding a number of paired analog measurements → it only increases the redundancy in the SE.
⇒ Implemented at the British Columbia Transmission Corporation

Phasor Assisted State Estimation - IV

Implementation in Power Industry SEs

⇒ Phasor angle - CSE [9], SDG& E [10]
⇒ Voltage and Current Phasors - NYPA [11], TVA [12]
⇒ Power Conversion - BCTC [13]

PMU Measurements

Utility	θ	$\tilde{V}$ (V, θ)	$\tilde{I}$ (I, δ)	S_{ij}	Total PMU Meas.
CSE [9]	23	—	—	—	23
SDG&E [14, 10]	$5^{\dagger}$	—	—	—	5
NYPA [11, 15]	—	10	24	—	34
TVA [16, 12]	—	$\star$	$\star$	—	18
BCTC [13]	—	—	—	$\triangle$	$\star$

Penetration of PMU-Measurements in Conventional SEs

Utility	SCADA Meas.	PMU Meas.	Total Meas	% SCADA	% PMU
CSE [9]	309	23	332	93.07	6.93
SDG&E [14, 10]	1800	5	1805	99.72	0.28
NYPA [11, 15]	850	34	884	96.15	3.85
TVA [16, 12]	17,000	18	17,018	99.89	0.11
BCTC [13]	$\star$	$\star$	—	—	—

Notation and Acronyms: $\star$ - Not available in literature, † - Implemented as angle differences, $\triangle$ - S_{ij} measurements derived from $\tilde{V}$ and $\tilde{I}$,$*$ 70% of the network is observable, CSE - *Sevillana de Electricidad*, SDG&E - San Diego Gas & Electric, NYPA - New York Power Authority, TVA - Tennessee Valley Authority, BCTC - British Columbia Transmission Corporation.

Additional Examples on Phadke and Thorp's Linear SE

Example 2: All $\tilde{V}$ and $\tilde{I}$ phasors measured except $\tilde{I}_{42m}$ - I

⇒ What happens to $\tilde{\boldsymbol{G}}$ when $\tilde{I}_{42m}$ is not measured?

⇒ Changes to the network-measurement matrices (from Example 1) are:

- $\boldsymbol{A}^T$: row 8 corresponding to $\tilde{I}_{42}$ is removed, the matrix is now (7×4)
- $\tilde{\boldsymbol{y}}$ loses row 8 and column 8 (diagonal entry $\tilde{y}_{42}$) becoming (7×7) and, $\tilde{\boldsymbol{y}}_{\boldsymbol{s}}$, and $\tilde{\boldsymbol{Y}}$ lose row 8 becoming (7×4)

⇒ White Gaussian Noise w/ `snr` of 75 dB is added to the loadflow solution.

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} 1.445 & -1.445 & 0 & 0 \\ -1.445 & 1.4609 & -0.0142 & -0.0017347 + j5.3651\times 10^{-6} \\ 0 & -0.0142 & 0.028404 & -0.0142 \\ 0 & -0.0017347 - j5.3651\times 10^{-6} & -0.0142 & 0.015992 \end{bmatrix}$$

⇒ Only one end of Line 2-4 is being measured, thus $\tilde{\boldsymbol{G}}$ is <u>symmetrical but complex</u>.

Solution

Voltage Magnitudes

Bus	$\lvert\tilde{V}^{\text{meas}}\rvert$	$\lvert\tilde{V}^{\text{se}}\rvert$	Residual
1	1.0502	1.0501	0.00013328
2	1.0274	1.0273	0.00012238
3	0.88639	0.88652	0.00013176
4	0.87869	0.87873	4.1387e-05

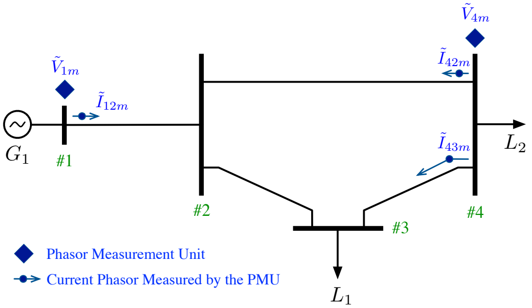
Voltage Angles

Bus	$\lvert\theta^{\text{meas}}\rvert$	$\lvert\theta^{\text{se}}\rvert$	Residual
1	19.9971	19.9817	0.015389
2	17.5119	17.4961	0.015811
3	-1.4645	-1.4826	0.018034
4	-1.4595	-1.4778	0.01833

Example 3: Measurements only at Buses 1 and 4 - I

⇒ Network-measurement matrices:

$$\boldsymbol{A}^T = \begin{matrix} \tilde{I}_{12} \\ \tilde{I}_{43} \\ \tilde{I}_{42} \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$



$$\tilde{\boldsymbol{y}}_{\boldsymbol{s}} = \begin{bmatrix} \tilde{y}_{12_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{y}_{43_0} \\ 0 & 0 & 0 & \tilde{y}_{42_0} \end{bmatrix}$$

$$\tilde{\boldsymbol{y}} = \mathrm{diag}([\tilde{y}_{12} \quad \tilde{y}_{43} \quad \tilde{y}_{42}])$$

$$\boldsymbol{U} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{R} = \mathrm{diag}([\sigma^2_{\tilde{V}_1} \quad \sigma^2_{\tilde{V}_4} \quad \sigma^2_{\tilde{I}_{12}} \quad \sigma^2_{\tilde{I}_{43}} \quad \sigma^2_{\tilde{I}_{42}}])$$

$$\tilde{\boldsymbol{Y}} = \begin{bmatrix} \tilde{y}_{12} + \tilde{y}_{12_0} & -\tilde{y}_{12} & 0 & 0 \\ 0 & 0 & -\tilde{y}_{43} & \tilde{y}_{43} + \tilde{y}_{43_0} \\ 0 & \tilde{y}_{42} & 0 & \tilde{y}_{42} + \tilde{y}_{42_0} \end{bmatrix}$$

⇒ White Gaussian Noise w/ `snr` of 75 dB is added to the loadflow solution.

Example 3: Measurements only at Buses 1 and 4 - II

⇒ Limited measurements are available only on one end of the transmission lines – the gain matrix is symmetric and complex

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} 0.7225 & -0.7225 & 0 & 0 \\ -0.7225 & 0.72429 & 0 & -0.0017 - j5.36 \times 10^{-6} \\ 0 & 0 & 0.0071 & -0.0071 - j5.36 \times 10^{-6} \\ 0 & -0.0017 + j5.36 \times 10^{-6} & -0.0071 + j5.36 \times 10^{-6} & 0.0087328 \end{bmatrix}$$

Solution

Voltage Magnitudes

Bus	$\lvert\tilde{V}^{meas}\rvert$	$\lvert\tilde{V}^{se}\rvert$	Residual
1	1.0506	1.0539	0.0033124
2	1.0277	1.0305	0.0028902
3	0.88631	0.89018	0.0038725
4	0.87919	0.88286	0.0036687

Voltage Angles

Bus	$\lvert\theta^{meas}\rvert$	$\lvert\theta^{se}\rvert$	Residual
1	20.0017	19.9319	0.06979
2	17.5144	17.4511	0.063315
3	-1.4628	-1.4622	0.00057954
4	-1.4636	-1.4632	0.00041215

Example 4 - Only $\tilde{V}$ Phasors Measured at all Buses

$\Rightarrow$ What happens to $\tilde{\boldsymbol{G}}$ when only $\tilde{V}$ are measured?
$\Rightarrow$ Changes to the network-measurement matrices (from Example 1) – there are no $\tilde{I}$ measurements, so $\boldsymbol{A}^T$, $\tilde{\boldsymbol{y}}$, $\tilde{\boldsymbol{y}}_{\boldsymbol{S}}$, and $\tilde{\boldsymbol{Y}}$; so the $\tilde{\boldsymbol{B}}$ is only formed by

$$\boldsymbol{U} = \text{diag}([1 \;\; 1 \;\; 1 \;\; 1])$$

$\Rightarrow$ The gain matrix is equal to the covariance matrix $\boldsymbol{R}$, thus real, symmetric, and diagonal. In this case, because we have set $\sigma_{\tilde{V}_i} = 0.00187$, the gain matrix is

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} 3.4969 \times 10^{-6} & 0 & 0 & 0 \\ 0 & 3.4969 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 3.4969 \times 10^{-6} & 0 \\ 0 & 0 & 0 & 3.4969 \times 10^{-6} \end{bmatrix}$$

Solution

Voltage Magnitudes

Bus	$\lvert\tilde{V}^{\text{true}}\rvert$	$\lvert\tilde{V}^{\text{meas}}\rvert$	$\lvert\tilde{V}^{\text{se}}\rvert$
1	1.05	1.0506	1.0506
2	1.0272	1.0277	1.0277
3	0.88654	0.88631	0.88631
4	0.87867	0.87919	0.87919

Voltage Angles

Bus	$\lvert\theta^{\text{true}}\rvert$	$\lvert\theta^{\text{meas}}\rvert$	$\lvert\theta^{\text{se}}\rvert$
1	20	20.0017	20.0017
2	17.5146	17.5144	17.5144
3	-1.4645	-1.4628	-1.4628
4	-1.4631	-1.4636	-1.4636