1. (12 points) There are three balls in an urn numbered 1,2,3. Hint: You should get different answers for each part below.

1.a. (4 points) You randomly select one ball from the urn. What is the smallest sample space for this problem?

Solution

\[ S = \{1,2,3\} \] since the only possible outcomes are 1, 2 and 3.

1.b. (4 points) You randomly select one ball from the urn. Then you put this ball aside and select a second ball from the urn. What is the smallest sample space for this problem?

Solution

Here, one outcome is an ordered pair where the 2\textsuperscript{nd} number is not the same as the first.

\[ S = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}. \]

1.c. (4 points) You repeat the experiment in part b, but before selecting the second ball from the urn, you put back the first ball and shake vigorously. What is the smallest sample space for this problem?

Solution

Here, one outcome is an ordered pair where the 2\textsuperscript{nd} number can be the same as the first.

\[ S = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}. \]

2. (8 points, 4 points each part) For part a and part b in question 1, list all possible events for that sample space. Again, you should get different answers in each part.

2.a. \( S = \{1,2,3\} \)

Solution

With 3 elements in \( S \), there should be \( 2^3 = 8 \) subsets of \( S \).

\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1\}, \{2\}, \{3\}, \emptyset.
2.b. \[ S = \{ (1,2), (1,3), (2,1), (2,3), (3,1), (3,2) \} . \]

**Solution**

With 6 elements in \( S \), there should be \( 2^6 = 64 \) subsets of \( S \).

One set with 6 elements: \( \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\} \)

6 sets with 5 elements: \( \{(1,2), (1,3), (2,1), (2,3), (3,1)\} , \{(1,2), (1,3), (2,1), (2,3), (3,2)\} , \{(1,2), (1,3), (2,1), (3,1), (3,2)\} , \{(1,2), (1,3), (2,3), (3,1), (3,2)\} , \{(1,2), (2,1), (2,3), (3,1)\} , \{(1,3), (2,1), (2,3), (3,1)\} . \)

15 sets with 4 elements: \( \{(1,2), (1,3), (2,1), (2,3)\} , \{(1,2), (1,3), (2,1), (3,1)\} , \{(1,2), (1,3), (2,3), (3,1)\} , \{(1,2), (1,3), (2,3), (3,2)\} , \{(1,2), (2,1), (2,3), (3,1)\} , \{(1,2), (2,1), (2,3), (3,2)\} , \{(1,2), (2,1), (3,1), (3,2)\} , \{(1,2), (2,3), (3,1), (3,2)\} , \{(1,3), (2,1), (2,3), (3,1)\} , \{(1,3), (2,1), (2,3), (3,2)\} , \{(1,3), (2,1), (3,1), (3,2)\} , \{(1,3), (2,3), (3,1), (3,2)\} , \{(2,1), (2,3), (3,1), (3,2)\} . \)

20 sets with 3 elements: \( \{(1,2), (1,3), (2,1)\} , \{(1,2), (1,3), (2,3)\} , \{(1,2), (1,3), (3,1)\} , \{(1,2), (2,1), (2,3)\} , \{(1,2), (2,1), (3,1)\} , \{(1,2), (2,1), (3,2)\} , \{(1,2), (2,3), (3,1)\} , \{(1,2), (2,3), (3,2)\} , \{(1,2), (3,1), (3,2)\} , \{(1,3), (2,1), (2,3)\} , \{(1,3), (2,1), (3,1)\} , \{(1,3), (2,1), (3,2)\} , \{(1,3), (2,3), (3,1)\} , \{(1,3), (2,3), (3,2)\} , \{(1,3), (3,1), (3,2)\} , \{(2,1), (2,3), (3,1)\} , \{(2,1), (2,3), (3,2)\} , \{(2,1), (3,1), (3,2)\} , \{(2,3), (3,1), (3,2)\} . \)

15 sets with 2 elements: \( \{(1,2), (1,3)\} , \{(1,2), (2,1)\} , \{(1,2), (2,3)\} , \{(1,2), (3,1)\} , \{(1,2), (3,2)\} , \{(1,3), (2,1)\} , \{(1,3), (2,3)\} , \{(1,3), (3,1)\} , \{(1,3), (3,2)\} , \{(2,1), (2,3)\} , \{(2,1), (3,1)\} , \{(2,1), (3,2)\} , \{(2,3), (3,1)\} , \{(2,3), (3,2)\} . \)

6 singletons (sets with one element): \( \{(1,2)\} , \{(1,3)\} , \{(2,1)\} , \{(2,3)\} , \{(3,1)\} , \{(3,2)\} . \)

1 set with no elements (the empty set): \( \emptyset . \)